AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA

LEAVING CERTIFICATE EXAMINATION, 2001

MATHEMATICS — HIGHER LEVEL

PAPER 2 (300 marks)

MONDAY, 11 JUNE — MORNING, 9.30 to 12.00

Attempt **FIVE** questions from Section **A** and **ONE** question from Section **B**.
Each question carries 50 marks.

WARNING: Marks may be lost if all necessary work is not clearly shown.

SECTION A

Answer FIVE questions from this section.

---

1. **(a)** A circle with centre $(-3, 7)$ passes through the point $(5, -8)$.
Find the equation of the circle.

(b) The equation of a circle is $(x+1)^2 + (y-8)^2 = 160$.
The line $x - 3y + 25 = 0$ intersects the circle at the points p and q.

(i) Find the co-ordinates of p and the co-ordinates of q.

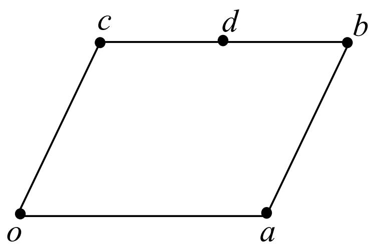
(ii) Investigate if $[pq]$ is a diameter of the circle.

(c) The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ passes through the points $(3, 3)$ and $(4, 1)$.
The line $3x - y - 6 = 0$ is a tangent to the circle at $(3, 3)$.

(i) Find the real numbers g, f and c.

(ii) Find the co-ordinates of the point on the circle at which the tangent parallel to $3x - y - 6 = 0$ touches the circle.

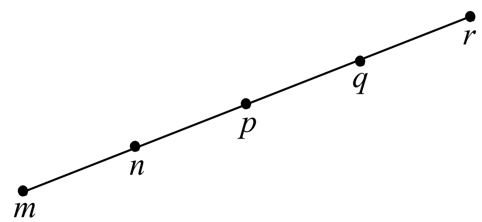
2. **(a)** *oabc* is a parallelogram where *o* is the origin.
d is the midpoint of $[cb]$.



(i) Express $\vec{b}$ in terms of $\vec{a}$ and $\vec{c}$.

(ii) Express $\vec{d}$ in terms of $\vec{a}$ and $\vec{c}$.

(b) $[mr]$ is divided into four line segments of equal length by the points n, p and q.



Given that $\vec{m} = -2\vec{i} + 3\vec{j}$ and $\vec{q} = 7\vec{i} - 9\vec{j}$, express

(i) $\vec{p}$ in terms of $\vec{i}$ and $\vec{j}$

(ii) $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

(c) *rst* is a triangle where $\vec{r} = -\vec{i} + 2\vec{j}$, $\vec{s} = -4\vec{i} - 2\vec{j}$ and $\vec{t} = 3\vec{i} - \vec{j}$.

(i) Express $\overrightarrow{rs}$, $\overrightarrow{st}$ and $\overrightarrow{tr}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Show that the triangle *rst* is right-angled at *r*.

(iii) Find the measure of $\angle rst$.

3. **(a)** The line B contains the points $(6,-2)$ and $(-4,10)$.
The line A with equation $ax+6y+21=0$ is perpendicular to B.

Find the value of the real number a.

(b) f is the transformation $(x,y) \rightarrow (x',y')$ where

$$x' = -5x - 6y$$
$$y' = 4x + 3y.$$

L is the line $x-9y=2$.

(i) Find the equation of $f(L)$, the image of L under f.

M is a line containing the point $(1,k)$ where $k \in \mathbf{Z}$.

(ii) Given that $f(M)$ is $5x'-2y'+3k=0$, find the value of k.

(c) N is the line $tx+(t-2)y+4=0$ where $t \in \mathbf{R}$.

(i) Write down the slope of N in terms of t.

(ii) Given that the angle between N and the line $x-3y+1=0$ is $45°$, find the two possible values of t.

4. **(a)** The length of an arc of a circle is 10 cm. The radius of the circle is 4 cm.

The measure of the angle at the centre of the circle subtended by the arc is θ.

(i) Find θ in radians.

(ii) Find θ in degrees, correct to the nearest degree.

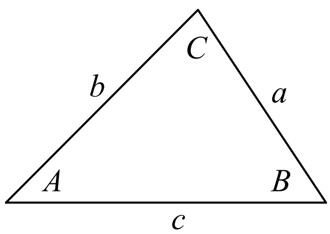
(b) **(i)** Write $\cos 2x$ in terms of $\sin x$.

(ii) Hence, find all the solutions of the equation

$$\cos 2x - \sin x = 1$$

in the domain $0° \leq x \leq 360°$.

(c) A triangle has sides a, b and c.
The angles opposite a, b and c are A, B and C, respectively.



(i) Prove that $a^2 = b^2 + c^2 - 2bc\cos A$.

(ii) Show that $c(b\cos A - a\cos B) = b^2 - a^2$.

5. **(a)** Evaluate $\lim_{\theta \to 0} \frac{\sin 7\theta}{\sin 2\theta}$.

(b) xyz is a triangle where $|xy| = 8$ cm and $|yz| = 6$ cm.

Given that the area of triangle xyz is 12 cm^2, find

(i) the two possible values of $|\angle xyz|$

(ii) the two possible values of $|xz|$, correct to one decimal place.

(c) A is an obtuse angle such that

$$\sin\left(A + \frac{\pi}{6}\right) + \sin\left(A - \frac{\pi}{6}\right) = \frac{4\sqrt{3}}{5}.$$

(i) Find $\sin A$ and $\tan A$.

(ii) Given that $\tan(A + B) = \frac{1}{2}$, find $\tan B$ and express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbf{Z}$ and $q \neq 0$.

6. **(a)** **(i)** How many different sets of three books or of four books can be selected from six different books?

(ii) How many of the above sets contain one particular book?

(b) Solve the difference equation

$$u_{n+2} - 8u_{n+1} + 11u_n = 0, \quad \text{where } n \geq 0,$$

given that $u_0 = 0$ and $u_1 = 2\sqrt{15}$.

(c) A box contains four silver coins, two gold coins and x copper coins.
Two coins are picked at random, and without replacement, from the box.

(i) Write down an expression in x for the probability that the two coins are copper.

If it is known that the probability of picking two copper coins is $\frac{4}{13}$,

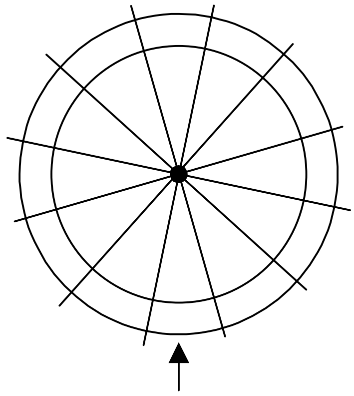
(ii) how many coins are in the box and

(iii) what is the probability that neither of the two coins picked is copper?

7. **(a)** **(i)** In how many different ways can four of the letters of the word FRIDAY be arranged if each letter is used no more than once in each arrangement?

(ii) How many of the above arrangements begin with the letter D and end with a vowel?

(b) To play a game a player spins a wheel.



The wheel is fixed to a wall. It spins freely around its centre point. Its rim is divided equally into twelve regions. Three of the regions are coloured red. Four are coloured blue. Five coloured green.

When the wheel stops an arrow fixed to the wall points to one of the regions. All the regions are equally likely to stop at the arrow. The colour of this region is the outcome of the game.

When the game is played twice, calculate the probability

that

(i) both outcomes are green

(ii) both outcomes are the same colour

(iii) the first outcome is red and the second is green

(iv) one outcome is green and the other is blue.

(c) Consider the numbers

$$1, \quad k, \quad 3k-2, \quad 9$$

where $k \in \mathbf{Z}$.

The mean of these numbers is $\overline{x}$. The standard deviation is σ.

(i) Express $\overline{x}$ in terms of k.

(ii) Given that $\sigma = \sqrt{20}$, find the value of k.

SECTION B

Answer ONE question from this section.

8. **(a)** Use integration by parts to find $\int x \cos x \, dx$.

(b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots$ is the Maclaurin series for $f(x)$.

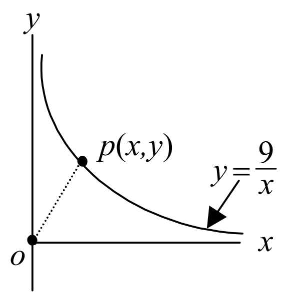
(i) Derive the Maclaurin series for $f(x) = \sin x$ up to and including the term containing x^7.

(ii) Write down the general term and use the Ratio Test to show that the series converges for all $x \in \mathbf{R}$.

(c) o is the origin, $(0,0)$.

$p(x, y)$ is a point on the curve $y = \frac{9}{x}$, where $x > 0$.

$|op|$ is the distance from the origin to p.



(i) Express $|op|$ in terms of x.

(ii) Given that there is one value of x for which $|op|$ is a minimum, find this value of x.

(iii) Hence, find the minimum value of $|op|$.

9. **(a)** Two fair dice are thrown.

(i) What is the probability of getting a four on both dice?
(ii) What is the probability of getting a four on at least one die?

(b) The probability of passing a driving test is $\frac{2}{3}$. Six students take the test.

Use a binomial distribution to find
(i) the probability that none of the students passes
(ii) the probability that half of the students pass the test.

(c) A particular drug gives relief from pain. The period of pain relief reported by people who are treated with the drug is normally distributed with mean 50 hours and standard deviation 16 hours.

In a random sample of 64 people who have been treated with the drug, what is the probability that the mean period of pain relief reported is between 48 hours and 53 hours?

10. **(a)** A binary operation $\circ$ is defined by $a \circ b = \dfrac{a+b}{2}$ where $a, b \in \mathbf{R}$.

Investigate if $(a \circ b) \circ c = a \circ (b \circ c)$.

(b) The group G, * is defined by the following Cayley table:

*	*e*	*a*	*b*	*c*	*d*	*f*	*g*	*h*
e	*e*	*a*	*b*	*c*	*d*	*f*	*g*	*h*
a	*a*	*b*	*d*	*h*	*e*	*c*	*f*	*g*
b	*b*	*d*	*e*	*g*	*a*	*h*	*c*	*f*
c	*c*	*h*	*g*	*b*	*f*	*a*	*e*	*d*
d	*d*	*e*	*a*	*f*	*b*	*g*	*h*	*c*
f	*f*	*c*	*h*	*a*	*g*	*e*	*d*	*b*
g	*g*	*f*	*c*	*e*	*h*	*d*	*b*	*a*
h	*h*	*g*	*f*	*d*	*c*	*b*	*a*	*e*

(i) Find the order of each element.

(ii) Write down three subgroups of order two.

(iii) H = $\{e, c, x, y\}$ is a subgroup of G. What elements of G do x and y represent?

(iv) Show that K = $\{e, b, f, h\}$ is a subgroup of G and explain why H and K are not isomorphic.

11. **(a)** h is the transformation $(x, y) \to (x', y')$ where $x' = 5x$ and $y' = 3y$.

(i) Find the image of the circle $x^2 + y^2 = 4$ under h.

(ii) Show that the image is an ellipse and find its eccentricity.

(b) Let g be a similarity transformation.

(i) The angle $\angle pqr$ is mapped to the angle $\angle p'q'r'$ under g.
Given that the line qs bisects $\angle pqr$, show that $q's'$ bisects $\angle p'q'r'$.

(ii) Hence, prove that if h is the incentre of the triangle pqr, $g(h)$ is the incentre of the triangle $\angle p'q'r'$.

(c) f is the transformation $(x, y) \to (x', y')$ where $x' = ax$ and $y' = by$ for $a > b > 0$.

(i) Given that $f(C)$ is the ellipse $\dfrac{x'^2}{a^2} + \dfrac{y'^2}{b^2} = 1$, show that C is the circle $x^2 + y^2 = 1$.

(ii) Hence, show that the locus of midpoints of parallel chords of the ellipse $f(C)$ is a diameter (less its endpoints) of $f(C)$.