Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2005

MATHEMATICS – HIGHER LEVEL

PAPER 2 (300 marks)

MONDAY, 13 JUNE – MORNING, 9:30 to 12:00

Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.
Each question carries 50 marks.

WARNING: **Marks will be lost if all necessary work is not clearly shown.**

Answers should include the appropriate units of measurement, where relevant.

SECTION A

Answer FIVE questions from this section.

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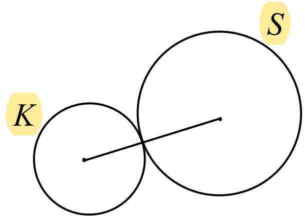
1. **(a)** Circles S and K touch externally.
Circle S has centre (8, 5) and radius 6.
Circle K has centre (2, −3).
Calculate the radius of K.

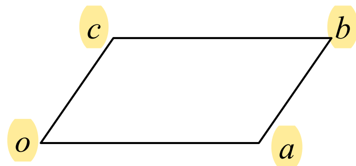


(b) **(i)** Prove that the equation of the tangent to the circle $x^2 + y^2 = r^2$ at the point (x_1, y_1) is $xx_1 + yy_1 = r^2$.

(ii) Hence, or otherwise, find the two values of b such that the line $5x + by = 169$ is a tangent to the circle $x^2 + y^2 = 169$.

(c) A circle passes through the points (7, 2) and (7, 10).
The line $x = -1$ is a tangent to the circle.
Find the equation of the circle.

2. **(a)** Copy the parallelogram $oabc$ into your answerbook.
Showing your work, construct the point d such that
$\vec{d} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} - \vec{c}$, where o is the origin.



(b) $\vec{p} = 3\vec{i} + 4\vec{j}$. $\vec{q}$ is the unit vector in the direction of $\vec{p}$.

(i) Express $\vec{q}$ and $\vec{q}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Express $11\vec{i} - 2\vec{j}$ in the form $k\vec{q} + l\vec{q}^{\perp}$, where $k, l \in \mathbf{R}$.

(c) $\vec{u} = \vec{i} + 5\vec{j}$ and $\vec{v} = 4\vec{i} + 4\vec{j}$.

(i) Find $\cos\angle uov$, where o is the origin.

(ii) $\vec{r} = (1-k)\vec{u} + k\vec{v}$, where $k \in \mathbf{R}$ and $k \neq 0$.
Find the value of k for which $|\angle uov| = |\angle vor|$.

3. **(a)** The line $L_1 : 3x - 2y + 7 = 0$ and the line $L_2 : 5x + y + 3 = 0$ intersect at the point p. Find the equation of the line through p perpendicular to L_2.

(b) The line K passes through the point $(-4, 6)$ and has slope m, where $m > 0$.

(i) Write down the equation of K in terms of m.

(ii) Find, in terms of m, the co-ordinates of the points where K intersects the axes.

(iii) The area of the triangle formed by K, the x-axis and the y-axis is 54 square units. Find the possible values of m.

(c) f is the transformation $(x, y) \rightarrow (x', y')$, where $x' = 3x - y$ and $y' = x + 2y$.

(i) Prove that f maps every pair of parallel lines to a pair of parallel lines. You may assume that f maps every line to a line.

(ii) $oabc$ is a parallelogram, where $[ob]$ is a diagonal and o is the origin. Given that $f(c) = (-1, 9)$, find the slope of ab.

4. **(a)** Evaluate $\lim\limits_{\theta \to 0} \dfrac{\sin 4\theta}{3\theta}$.

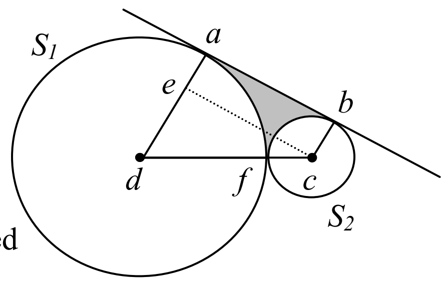
(b) **(i)** Using $\cos 2A = \cos^2 A - \sin^2 A$, or otherwise,

prove $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$.

(ii) Hence, or otherwise, solve the equation

$1 + \cos 2x = \cos x$, where $0^\circ \leq x \leq 360^\circ$.

(c) S_1 is a circle of radius 9 cm and S_2 is a circle of radius 3 cm. S_1 and S_2 touch externally at f. A common tangent touches S_1 at point a and S_2 at b.

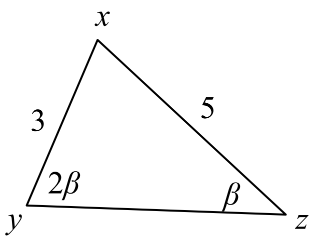


(i) Find the area of the quadrilateral $abcd$. Give your answer in surd form.

(ii) Find the area of the shaded region, which is bounded by $[ab]$ and the minor arcs af and bf.

5. **(a)** The area of an equilateral triangle is $4\sqrt{3}$ cm^2.
Find the length of a side of the triangle.

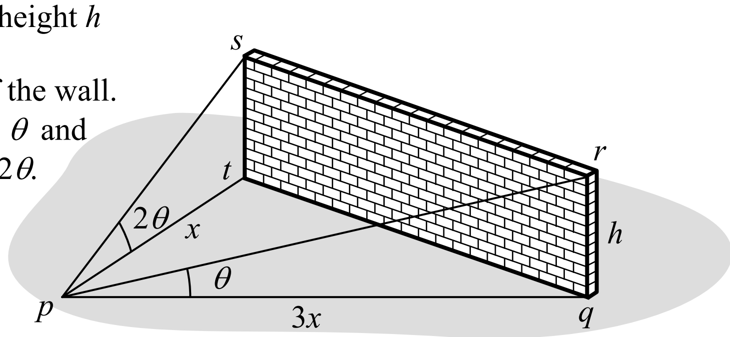
(b) In the triangle xyz, $|\angle xyz| = 2\beta$ and $|\angle xzy| = \beta$.
$|xy| = 3$ and $|xz| = 5$.



(i) Use this information to express $\sin 2\beta$ in the form $\frac{a}{b}\sin\beta$, where $a, b \in \mathbf{N}$.

(ii) Hence express $\tan\beta$ in the form $\frac{\sqrt{c}}{d}$, where $c, d \in \mathbf{N}$.

(c) $qrst$ is a vertical rectangular wall of height h on level ground.
p is a point on the ground in front of the wall.
The angle of elevation of r from p is θ and the angle of elevation of s from p is 2θ.
$|pq| = 3|pt|$.
Find θ.



6. **(a)** How many three-digit numbers can be formed from the digits 1, 2, 3, 4, 5, if

(i) the three digits are all different

(ii) the three digits are all the same?

(b) **(i)** Solve the difference equation $u_{n+2} - 4u_{n+1} - 8u_n = 0$, where $n \geq 0$, given that $u_0 = 0$ and $u_1 = 2$.

(ii) Verify that your solution gives the correct value for u_2.

(c) Nine cards are numbered from 1 to 9. Three cards are drawn at random from the nine cards.

(i) Find the probability that the card numbered 8 is not drawn.

(ii) Find the probability that all three cards drawn have odd numbers.

(iii) Find the probability that the sum of the numbers on the cards drawn is greater than the sum of the numbers on the cards not drawn.

7. **(a)** **(i)** How many different groups of four can be selected from five boys and six girls?

(ii) How many of these groups consist of two boys and two girls?

(b) There are sixteen discs in a board-game: five blue, three green, six red and two yellow. Four discs are chosen at random. What is the probability that

(i) the four discs are blue

(ii) the four discs are the same colour

(iii) all four discs are different in colour

(iv) two of the discs are blue and two are not blue?

(c) On 1st September 2003 the mean age of the first-year students in a school is 12.4 years and the standard deviation is 0.6 years. One year later all of these students have moved into second year and no other students have joined them.

(i) State the mean and the standard deviation of the ages of these students on 1st September 2004. Give a reason for each answer.

A new group of first-year students begins on 1st September 2004. This group has a similar age distribution and is of a similar size to the first-year group of September 2003.

(ii) State the mean age of the combined group of the first-year and second-year students on 1st September 2004.

(iii) State whether the standard deviation of the ages of this combined group is less than, equal to, or greater than 0.6 years. Give a reason for your answer.

SECTION B

Answer ONE question from this section.

8. **(a)** Use integration by parts to find $\int x^2 \ln x dx$.

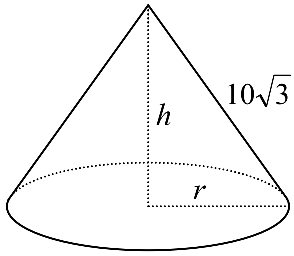
(b) **(i)** Derive the Maclaurin series for $f(x) = \ln(1+x)$ up to and including the term containing x^3.

(ii) Use those terms to find an approximation for $\ln\frac{11}{10}$.

(iii) Write down the general term of the series $f(x)$ and hence show that the series converges for $-1 < x < 1$.

(c) A cone has radius r cm, vertical height h cm and slant height $10\sqrt{3}$ cm.

Find the value of h for which the volume is a maximum.



9. **(a)** z is a random variable with standard normal distribution. Find $P(1 < z < 2)$.

(b) During a match John takes a number of penalty shots. The shots are independent of each other and his probability of scoring with each shot is $\frac{4}{5}$.

(i) Find the probability that John misses each of his first four penalty shots.

(ii) Find the probability that John scores exactly three of his first four penalty shots.

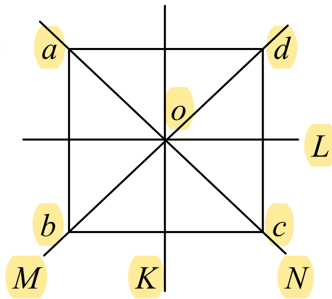
(iii) If John takes ten penalty shots during the match, find the probability that he scores at least eight of them.

(c) A survey was carried out to find the weekly rental costs of holiday apartments in a certain country. A random sample of 400 apartments was taken. The mean of the sample was €320 and the standard deviation was €50.

Form a 95% confidence interval for the mean weekly rental costs of holiday apartments in that country.

10. **(a)** Show that $\{0, 2, 4\}$ forms a group under addition modulo 6. You may assume associativity.

(b) R_{90° and S_M are elements of D_4, the dihedral group of a square.

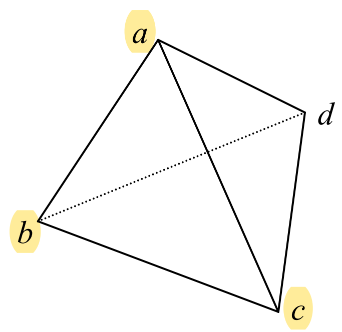


(i) List the other elements of the group.

(ii) Find $C(S_M)$, the centralizer of S_M.

(c) A regular tetrahedron has twelve rotational symmetries.
These form a group under composition.
The symmetries can be represented as permutations of the vertices a, b, c and d.

$X = \left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ b & a & d & c \end{pmatrix} \right\}, \circ$ is a subgroup of this tetrahedral group.



(i) Write down one other subgroup of order 2.

(ii) Write down a subgroup of order 3.

(iii) Write down the only subgroup of order four.

11. **(a)** Find the equation of an ellipse with centre (0, 0), eccentricity $\frac{5}{6}$ and one focus at (10, 0).

(b) f is a similarity transformation having magnification ratio k.
A triangle abc is mapped onto a triangle $a'b'c'$ under f.
Prove that $|\angle abc| = |\angle a'b'c'|$.

(c) g is the transformation $(x, y) \to (x', y')$, where $x' = ax$ and $y' = by$ and $a > b > 0$.

(i) C is the circle $x^2 + y^2 = 1$. Show that $g(C)$ is an ellipse.

(ii) L and K are tangents at the end points of a diameter of the ellipse $g(C)$.
Prove that L and K are parallel.

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