Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2007

MATHEMATICS – HIGHER LEVEL

PAPER 2 (300 marks)

MONDAY, 11 JUNE – MORNING, 9:30 to 12:00

Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.
Each question carries 50 marks.

WARNING: **Marks will be lost if all necessary work is not clearly shown.**

Answers should include the appropriate units of measurement, where relevant.

SECTION A

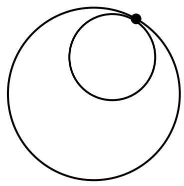
Answer FIVE questions from this section.

1. **(a)** The following parametric equations define a circle:

$$x = 5 + 7\cos\theta, \quad y = 7\sin\theta, \text{ where } \theta \in \mathbf{R}.$$

What is the Cartesian equation of the circle?

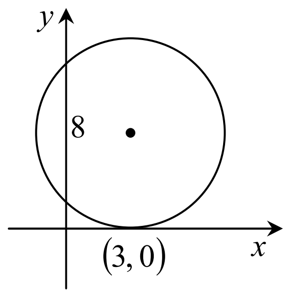
(b) $x^2 + y^2 - 4x - 6y + 5 = 0$ and $x^2 + y^2 - 6x - 8y + 23 = 0$ are two circles.



(i) Prove that the circles touch internally.

(ii) Find the coordinates of the point of contact of the two circles.

(c) A circle has its centre in the first quadrant.
The x-axis is a tangent to the circle at the point (3, 0).
The circle cuts the y-axis at points that are 8 units apart.
Find the equation of the circle.



2. **(a)** $\vec{x} = -2\vec{i} + 5\vec{j}$ and $\overrightarrow{xy} = -6\vec{i} - 8\vec{j}$. Express $\vec{y}$ in terms of $\vec{i}$ and $\vec{j}$.

(b) $\vec{a} = 5\vec{i}$ and $\vec{b} = \sqrt{3}\,\vec{i} + 3\vec{j}$.

(i) Show that $\overrightarrow{ab}$ is not perpendicular to $\vec{b}$.

(ii) Find the value of the real number k, given that $\vec{c} = k\vec{b}$ and $\overrightarrow{ac} \perp \vec{b}$.

(c) $\vec{p} = 3\vec{i} + 4\vec{j}$ and $\vec{q} = 5\vec{i} + 12\vec{j}$.

$$\vec{r} = \frac{65t}{16}\left(\frac{\vec{p}}{|\vec{p}|} + \frac{\vec{q}}{|\vec{q}|}\right), \text{ where } t > 0.$$

(i) Express $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find $\vec{p}.\vec{r}$ and $\vec{q}.\vec{r}$.

(iii) Hence, show that r is on the bisector of $\angle poq$, where o is the origin.

3. **(a)** Find the area of the triangle with vertices $(1,1)$, $(8,-5)$ and $(5,-2)$.

(b) f is the transformation $(x,y) \to (x',y')$, where $x' = 4x + 2y$ and $y' = -3x - y$.
K is the line $x + y = 0$.

(i) Show that K is its own image under f.

(ii) $p\ (1, -1)$ and $q\ (3, -3)$ are two points.
Find the ratio $|pq| : |f(p)f(q)|$, giving your answer in its simplest form.

(c) Consider the equation $k(3x - 5y + 6) + l(5x - 7y + 4) = 0$, where $k, l \in \mathbf{R}$.

(i) Show that for all k and l, the given equation represents a line passing through the point of intersection of $3x - 5y + 6 = 0$ and $5x - 7y + 4 = 0$.

(ii) Find the relationship between k and l for which the given equation represents a line of slope 2.

(iii) If $k = 1$, what line through the point of intersection cannot be represented by the given equation? Justify your answer.

4. **(a)** Show that $(\cos A + \sin A)^2 = 1 + \sin 2A$.

(b) Find all the solutions of the equation

$$6\cos^2 x + \sin x - 5 = 0, \text{ where } 0^\circ \le x \le 360^\circ.$$

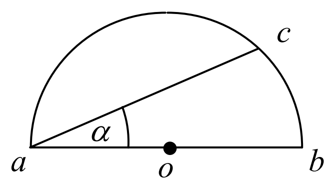
Give the solutions correct to the nearest degree.

(c) $[ab]$ is the diameter of a semicircle of centre o and radius-length r.
$[ac]$ is a chord such that $|\angle cab| = \alpha$, where α is in radian measure.

(i) Find $|ac|$ in terms of r and α.

(ii) $[ac]$ bisects the area of the semicircular region.

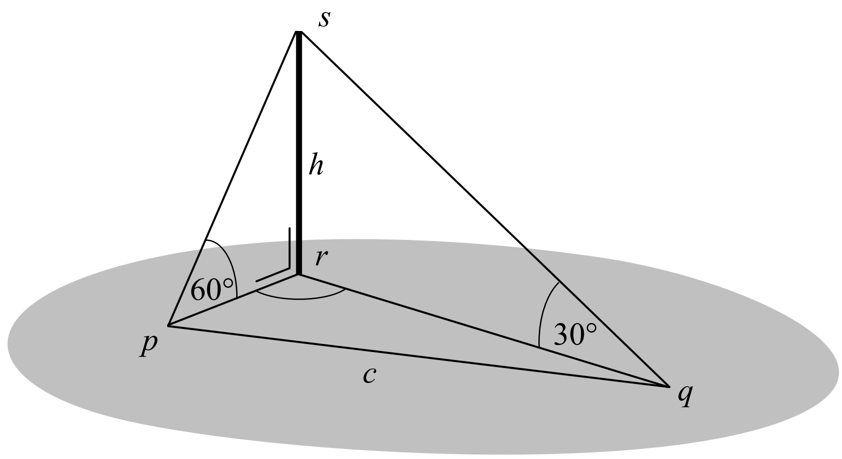
Show that $2\alpha + \sin 2\alpha = \dfrac{\pi}{2}$.



5. **(a)** Evaluate $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 3x}$.

(b) Using the formula $\cos(A+B)=\cos A\cos B-\sin A\sin B$, derive a formula for $\cos(A-B)$ and hence prove that $\sin(A+B)=\sin A\cos B+\cos A\sin B$.

(c) p, q and r are three points on horizontal ground.
$[sr]$ is a vertical pole of height h metres.
The angle of elevation of s from p is 60° and the angle of elevation of s from q is 30°.
$|pq| = c$ metres.
Given that $3c^2 = 13h^2$, find $|\angle prq|$.



6. **(a)** Six people, including Mary and John, sit in a row.

(i) How many different arrangements of the six people are possible?

(ii) In how many of these arrangements are Mary and John next to each other?

(b) α and β are the roots of the quadratic equation $px^2 + qx + r = 0$.
$u_n = l\alpha^n + m\beta^n$, for all $n \in \mathbf{N}$.
Show that $pu_{n+2} + qu_{n+1} + ru_n = 0$, for all $n \in \mathbf{N}$.

(c) w white discs and r red discs are placed in a box. Two of the discs are drawn at random from the box. The probability that both discs are red is p.

(i) Find p in terms of w and r.

(ii) When $w = 1$, find the value of r for which $p = \frac{1}{2}$.

(iii) There are other values of w and r that also give $p = \frac{1}{2}$.

The next smallest such value of w is even.

By investigating the even numbers in turn, find this value of w and the corresponding value of r.

7. **(a)** **(i)** How many different selections of four letters can be made from the letters of the word FLORIDA ?

(ii) How many of these selections contain at least one vowel?

(b) Two dice are thrown.

(i) What is the probability of getting two identical numbers or a total of five?

(ii) What is the probability that the product of the two numbers thrown is at least twice their sum?

(c) **(i)** Find, in terms of a and d, the mean of the first seven terms of an arithmetic sequence with first term a and common difference d.

(ii) Show that the standard deviation of these seven numbers is $2d$.

SECTION B

Answer ONE question from this section.

8. **(a)** p and q are real numbers such that $p + q = 1$.
Find the value of p that maximizes the product pq.

(b) **(i)** Derive the Maclaurin series for $f(x) = (1+x)^m$ up to and including the term containing x^3.

(ii) Given that the general term of the series $f(x)$ is

$$\frac{m(m-1)(m-2).........(m-r+1)}{r!}x^r,$$

show that the series converges for $-1 < x < 1$.

(c) Evaluate $\displaystyle\int_0^1 \tan^{-1} x \, dx.$

9. **(a)** Two events E_1 and E_2 are independent. $P(E_1) = \frac{1}{5}$ and $P(E_2) = \frac{1}{7}$. Find

(i) $P(E_1 \cap E_2)$

(ii) $P(E_1 \cup E_2)$.

(b) Five unbiased coins are tossed.

(i) Find the probability of getting three heads and two tails.

(ii) The five coins are tossed eight times. Find the probability of getting three heads and two tails exactly four times.
Give your answer correct to three decimal places.

(c) The amounts due on monthly mobile phone bills are normally distributed with mean €53 and standard deviation €15.

(i) If a bill is chosen at random, find the probability that the amount due is between €47 and €74.

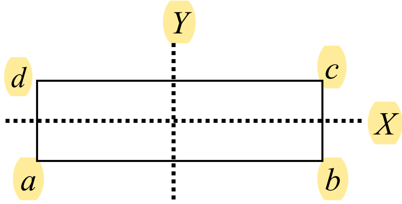
(ii) A random sample of 900 bills is taken. Find the probability that the mean amount due on the bills in the sample is greater than €53·30.

10. **(a)** For each of the following, give a reason why it is not a group.

(i) The set of natural numbers under subtraction.

(ii) The set of real numbers under multiplication.

(b) $G = \{I_\pi, R_{180°}, S_X, S_Y\}$ is the set of symmetries of the rectangle *abcd*.



(i) Show that G is a group under composition.
You may assume that composition of symmetries is associative.

(ii) Find $Z(G)$, the centre of the group.

(c) Use Lagrange's theorem to prove that

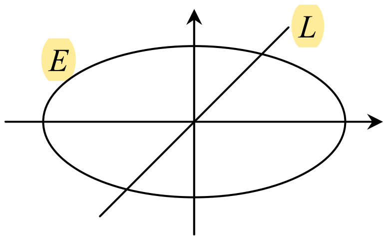
(i) any group of prime order is cyclic.

(ii) the order of any element of a finite group G divides the order of G.

11. **(a)** Find the eccentricity of the ellipse with equation $\frac{x^2}{64}+\frac{y^2}{48}=1$.

(b) Prove that a similarity transformation maps the orthocentre of a triangle onto the orthocentre of the image of the triangle.

(c) E is the ellipse $\frac{x^2}{4}+y^2=1$ and L is the line $y = x$.
Using a transformation that maps E to the unit circle, or otherwise, find the equation of the diameter that is conjugate to L in E.



Blank Page