Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2011

Mathematics (Project Maths – Phase 2)

Paper 1

Higher Level

Friday 10 June Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	100 marks	4 questions
Section B	Contexts and Applications	100 marks	2 questions
Section C	Functions and Calculus (old syllabus)	100 marks	3 questions

Answer questions as follows:

In Section A, answer **all four** questions

In Section B, answer **both** Question 5 **and** Question 6

In Section C, answer **any two** of the three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 100 marks

Answer **all four** questions from this section.

Question 1 **(25 marks)**

(a) Explain what it means to say that $\sqrt{3}$ is not a rational number.

(b) Given a line segment of length one unit, show clearly how to construct a line segment of length $\sqrt{3}$ units, using only a compass and straight edge.

(c) Solve the equation $x^2 - 2\sqrt{3}x - 9 = 0$, giving your answers in the form $a\sqrt{3}$, where $a \in \mathbb{Q}$.

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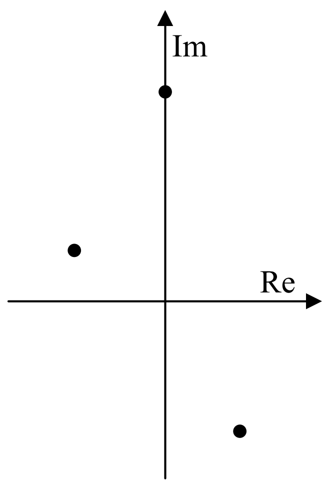
Question 2 **(25 marks)**

(a) **(i)** Write the complex number $1-i$ in polar form.

(ii) Use De Moivre's theorem to evaluate $(1-i)^9$, giving your answer in rectangular form.

(b) A complex number z has modulus greater than 1.
The three numbers z, z^2, and z^3 are shown on the Argand diagram.
One of them lies on the imaginary axis, as shown.



(i) Label the points on the diagram to show which point corresponds to which number.

(ii) Find θ, the argument of z.

Question 3 **(25 marks)**

(a) The cubic function $f : x \mapsto x^3 + 7x^2 + 17x + 15$ has one integer root and two complex roots. Find all three roots.

(b) Using part **(a)**, or otherwise, solve the equation $(x-2)^3 + 7(x-2)^2 + 17(x-2) + 15 = 0$.

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Question 4 **(25 marks)**

In a science experiment, a quantity $Q(t)$ was observed at various points in time t. Time is measured in seconds from the instant of the first observation. The table below gives the results.

t	0	1	2	3	4
$Q(t)$	2·920	2·642	2·391	2·163	1·957

Q follows a rule of the form $Q(t) = Ae^{-bt}$, where A and b are constants.

(a) Use any two of the observations from the table to find the value of A and the value of b, correct to three decimal places.

(b) Use a different observation from the table to verify your values for A and b.

(c) Show that $Q(t)$ is a constant multiple of $Q(t-1)$, for $t \geq 1$.

(d) Find the value of the constant k for which $Q(t+k) = \frac{1}{2}Q(t)$, for all $t \geq 0$.

Give your answer correct to two decimal places.

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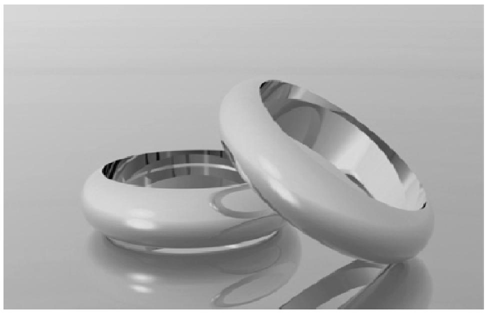
Section B	Contexts and Applications	100 marks

Answer **both** Question 5 **and** Question 6.

Question 5 **(50 marks)**

Gold jewellery is made from a gold alloy – that is, a mixture of pure gold and other metals. The purity of the material is measured by its "carat rating", given by the formula



$$c = \frac{24m_g}{m_t}$$

where c = carat rating
m_g = mass of gold in the material
m_t = total mass of the material.

A jeweller is recycling old gold jewellery. He has the following old jewellery in stock:
147 grams of 9-carat gold
85 grams of 18-carat gold.

He can melt down this old jewellery and mix it in various proportions to make new jewellery of different carat values. The value of the old jewellery is equal to the value of its gold content only. Gold is valued at €36 per gram.

(a) What is the total value of the jeweller's stock of old jewellery?

(b) The jeweller wants to make a 15-carat gold pendant weighing 21 grams. He melts down some 9-carat gold and some 18-carat gold to do this. How many grams of each should he use in order to get the 21 grams of 15-carat gold?

(c) The other metals in the gold alloy are copper and silver. The colour of the alloy depends on the ratio of copper to silver. In all of the old jewellery, the amount of silver is equal to the amount of copper. The jeweller has a stock of pure silver that he can add to any mixture. He wants to make an item that:

- weighs 48 grams
- is of 15-carat gold purity
- has twice as much silver as copper.

(i) How many grams of copper will this item contain?

(ii) How many grams of each type of stock (9-carat gold, 18-carat gold, and pure silver) should the jeweller use in order to make this item?

There is some more space for work on the next page.

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(d) A large jewellery business makes and sells 14-carat gold wedding rings, weighing an average of 5 grams each. The cost of producing each ring is €135 plus the value of the gold. The manager has noted that the more they charge for the rings, the fewer they sell. In particular:

- if they charge €200, they sell an average of twenty per month
- for each additional €20 charged, the number sold drops by one per month.

(i) Taking the price charged as €$(200 + 20x)$, find an expression in x for the monthly profit from these rings.

(ii) Find the range of selling prices for which the monthly profit is at least €1600.

Question 6 **(50 marks)**

Most lottery games in the USA allow winners of the jackpot prize to choose between two forms of the prize: an *annual-payments* option or a *cash-value* option. In the case of the *New York Lotto*, there are 26 annual payments in the *annual-payments* option, with the first payment immediately, and the last payment in 25 years' time. The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The *cash-value* option pays a smaller amount than this.

(a) If the amount of the first annual payment is A, write down, in terms of A, the amount of the second, third, fourth and 26th payments.

1st payment (now): $\underline{\quad A \quad}$

2nd payment: ______________

3rd payment: ______________

4th payment: ______________

$\vdots$ $\vdots$

26th payment: ______________

(b) The 26 payments form a geometric series. Use this fact to express the advertised jackpot prize in terms of A.

(c) Find, correct to the nearest dollar, the value of A that corresponds to an advertised jackpot prize of \$21·5 million.

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(d) A winner who chooses the *cash-value* option receives, immediately, the total of the present values of the 26 annual payments. The interest rate used for the present-value calculations is 4·78%. We want to find the cash value of the prize referred to in part **(c)**.

(i) Complete the table below to show the actual amount and the present value of each of the first three annual payments.

payment number	time to payment (years)	actual amount	present value
1	0		
2	1		
3	2		

(ii) Write down, in terms of n, an expression for the present value of the nth annual payment.

(iii) Find the amount of prize money payable under the *cash-value* option. That is, find the total of the present values of the 26 annual payments.

Give your answer in millions, correct to one decimal place.

(e) The jackpot described in parts **(c)** and **(d)** above was won by an Irish woman earlier this year. She chose the *cash-value* option. After tax, she received \$7·9 million. What percentage of tax was charged on her winnings?

Section C	Functions and Calculus (old syllabus)	100 marks

Answer **any two** of the three questions from this section.

Question 7 **(50 marks)**

(a) Differentiate $\cos^2 x$ with respect to x.

(b) The equation of a curve is $y = e^{-x^2}$.

(i) Find $\dfrac{dy}{dx}$.

(ii) Find the co-ordinates of the turning point of the curve.

(iii) Determine whether this turning point is a local maximum or a local minimum.

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(c) The function f is defined as $x \to \dfrac{2x}{x+1}$, where $x \in \mathbb{R} \setminus \{-1\}$.

(i) Find the equations of the asymptotes of the curve $y = f(x)$.

(ii) P and Q are distinct points on the curve $y = f(x)$.
The tangent at Q is parallel to the tangent at P.
The co-ordinates of P are (1, 1).
Find the co-ordinates of Q.

(iii) Verify that the point of intersection of the asymptotes is the midpoint of $[PQ]$.

Question 8 (50 marks)

(a) Find the slope of the tangent to the curve $x^2 + y^3 = x - 2$ at the point (3, –2).

(b) A curve is defined by the parametric equations

$$x = \frac{t-1}{t+1} \quad \text{and} \quad y = \frac{-4t}{(t+1)^2}, \text{ where } t \neq -1.$$

(i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

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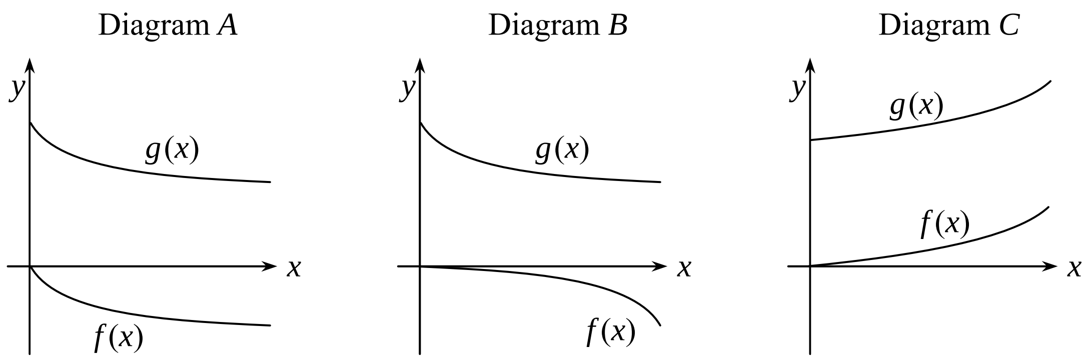
(ii) Hence find $\frac{dy}{dx}$, and express your answer in terms of x.

(c) The functions f and g are defined on the domain $x \in \mathbb{R} \setminus \{-1, 0\}$ as follows:

$f : x \to \tan^{-1}\left(\frac{-x}{x+1}\right)$ and $g : x \to \tan^{-1}\left(\frac{x+1}{x}\right)$.

(i) Show that $f'(x) = \frac{-1}{2x^2 + 2x + 1}$.

(ii) It can be shown that $f'(x) = g'(x)$.
One of the three diagrams A, B, or C below represents parts of the graphs of f and g. Based only on the derivatives, state which diagram is the correct one, and state also why each of the other two diagrams is incorrect.



Question 9 **(50 marks)**

(a) Find $\int \left(x^3 + \sqrt{x}\right) dx$.

(b) **(i)** Evaluate $\displaystyle\int_0^2 \frac{x+1}{x^2+2x+2}\, dx$.

(ii) Evaluate $\displaystyle\int_0^2 \frac{x^2+2x+2}{x+1}\, dx$.

(c) Use integration methods to establish the formula $A = \pi r^2$ for the area of a disc of radius r.

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