Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2011

Mathematics
(Project Maths – Phase 2)

Paper 2

Higher Level

Monday 13 June Morning 9:30 – 12:00

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	2 questions

Answer **all eight** questions, as follows:

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Question 7 and Question 8.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) A random variable X follows a normal distribution with mean 20 and standard deviation 5. Find $P(14 \leq X \leq 26)$.

(b) There are 16 girls and 8 boys in a class. Half of these 24 students study French. The probability that a randomly selected girl studies French is 1·5 times the probability that a randomly selected boy studies French. How many of the boys in the class study French?

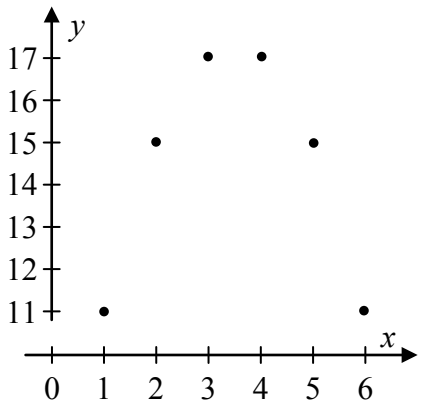
Question 2 **(25 marks)**

(a) Explain, with the aid of an example, what is meant by the statement:

"Correlation does not imply causality."

(b) The data given in the table below and represented in the scatter diagram are pairs of observations of the variables x and y.

x	1	2	3	4	5	6
y	11	15	17	17	15	11



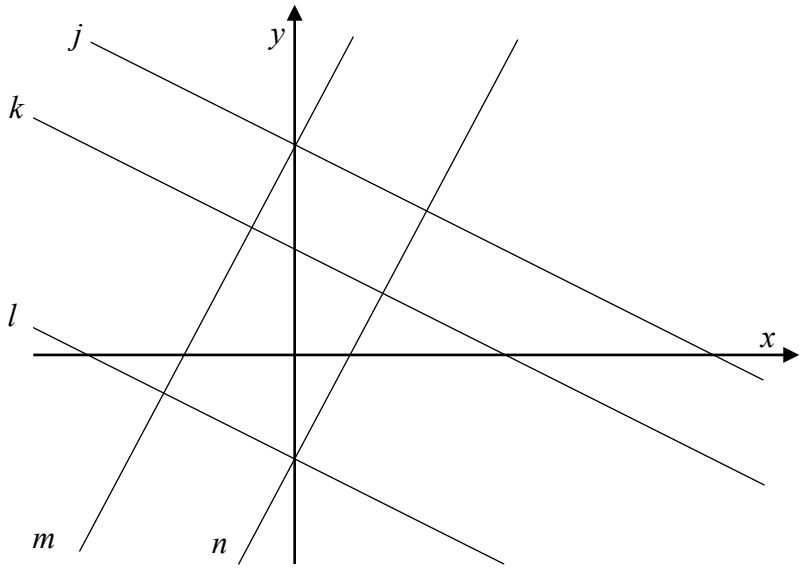
(i) Calculate the correlation coefficient.

Answer: _____

(ii) What kind of relationship, if any, do the observed data suggest exists between x and y?

Question 3 **(25 marks)**

In the co-ordinate diagram shown, the lines j, k, and l are parallel, and so are the lines m and n. The equations of four of the five lines are given in the table below.



Equation	Line
$x+2y=-4$	
$2x-y=-4$	
$x+2y=8$	
$2x-y=2$	

(a) Complete the table, by matching four of the lines to their equations.

(b) Hence, insert scales on the x-axis and y-axis.

(c) Hence, find the equation of the remaining line, given that its x-intercept and y-intercept are both integers.

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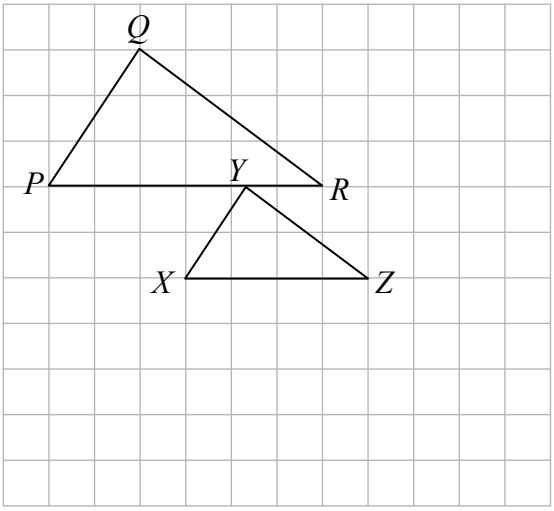
Question 4 **(25 marks)**

Two triangles are drawn on a square grid as shown. The points P, Q, R, X, and Z are on vertices of the grid, and the point Y lies on $[PR]$. The triangle PQR is an enlargement of the triangle XYZ.



(a) Calculate the scale factor of the enlargement, showing your work.

(b) By construction or otherwise, locate the centre of enlargement on the diagram above.

(c) Calculate $|YR|$ in grid units.

Question 5 **(25 marks)**

The line $x+3y=20$ intersects the circle $x^2+y^2-6x-8y=0$ at the points P and Q.
Find the equation of the circle that has $[PQ]$ as diameter.

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Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:

Given:

To prove:

Construction:

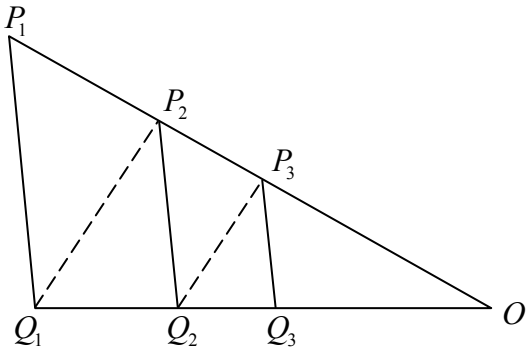
Proof:

OR

Question 6B

In the diagram, P_1Q_1, P_2Q_2, and P_3Q_3 are parallel and so also are Q_1P_2 and Q_2P_3.

Prove that $|P_1Q_1| \times |P_3Q_3| = |P_2Q_2|^2$.



Section B — Contexts and Applications — 150 marks

Answer Question 7 and Question 8.

Question 7 (75 marks)

(a) Some students are using a database of earthquakes to investigate the times between the occurrences of serious earthquakes around the world. They extract information about all of the earthquakes in the 20th century that caused at least 1000 deaths. There are 115 of these.

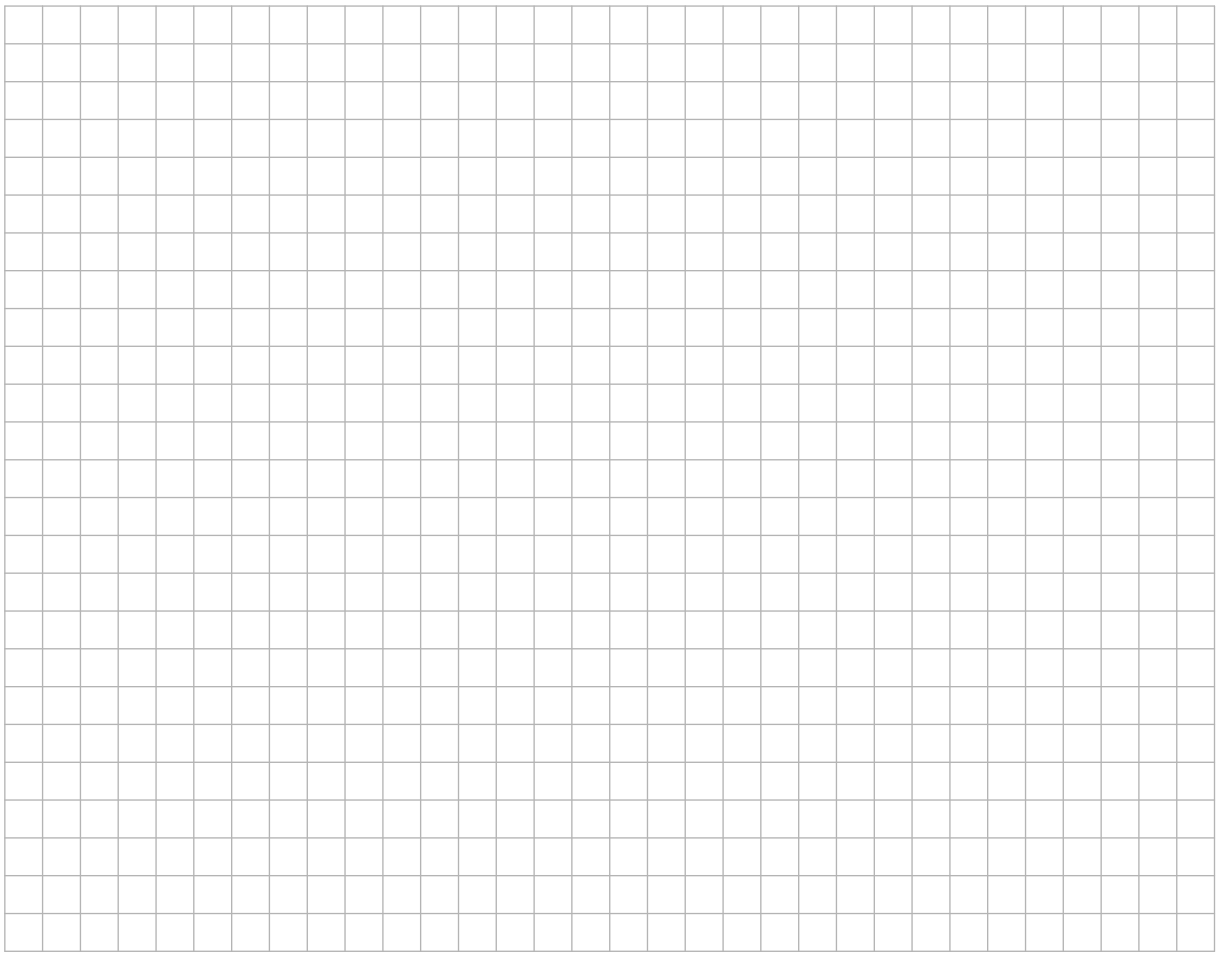
The students wonder whether there are patterns in the timing of these earthquakes, so they look at the number of days between each successive pair of these earthquakes.

They make the following table, showing the number of earthquakes for which the time interval from the previous earthquake is as shown.

Time in days from previous earthquake	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600	600 – 700	700 – 800	800 – 1000	1000 – 1300
Number of earthquakes	31	24	12	14	8	7	5	6	5	3

[Source: National geophysical data center, significant earthquake database: www.ngdc.noaa.gov]

(i) Create a suitable graphical representation of the distribution.



(ii) Describe the distribution. Your description should refer to the shape of the distribution and should include an estimate of the median.

(iii) The mean time between these earthquakes is 309 days and the standard deviation is 277 days. Suppose that such an earthquake has just occurred and that we want to find the probability that the time to the next one will be between 100 and 200 days. Explain why it would **not** be correct to use standard normal distribution tables (z-tables) to do this.

(iv) Based on the information presented in this question so far, what is the best estimate for the probability described in part **(iii)** above? Explain your reasoning.

(v) As stated at the beginning, the students chose to analyse earthquake timings by looking at the time intervals between the occurrences of a particular type of earthquake. Suggest a different way that they could have looked at the data in the database in order to try to find patterns in the timing of earthquakes.

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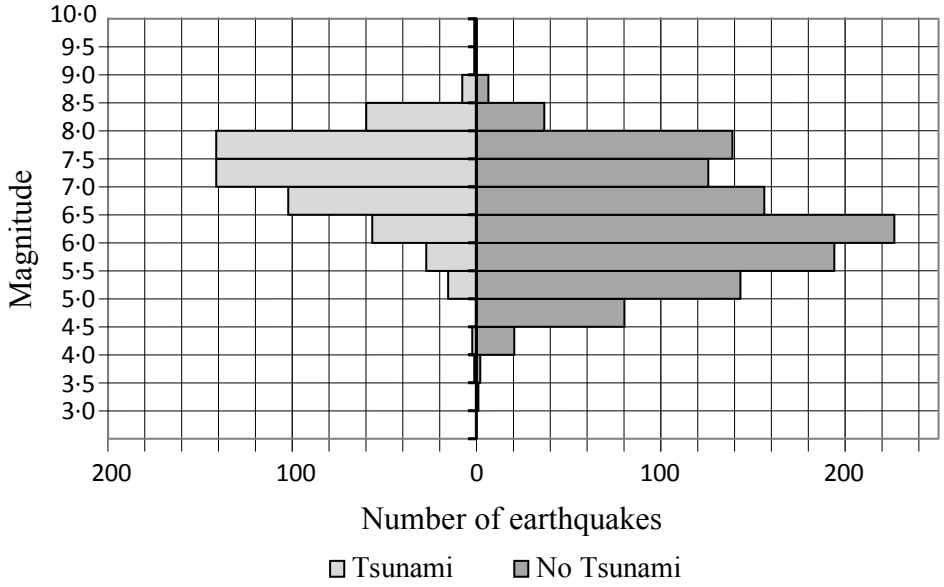
(b) The students heard a reporter saying that "strong earthquakes will cause large destructive ocean waves called tsunamis, while weaker ones will not". They decide to check this. They draw two histograms back to back, one showing the magnitudes of the earthquakes that caused tsunamis, and the other showing the magnitudes of those that did not. They use all of the suitable data from the 20th century that were recorded in this particular database.



(i) Comment on the reporter's statement, using information from the diagram to support your answer, and suggest a more accurate statement.

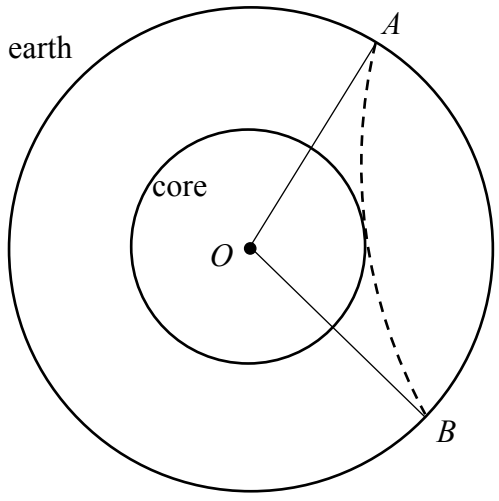
(ii) By taking suitable readings from the diagram, estimate the probability that an earthquake of magnitude between 6·5 and 7·0 will cause a tsunami.

(iii) Consider the next six earthquakes of magnitude at least 7·5. Find an estimate for the probability that at least four of them will cause a tsunami, assuming that these six events are independent of each other.

(c) Scientists use information about seismic waves from earthquakes to find out about the internal structure of the earth.

The diagram below represents a circular cross-section of the earth. The dashed curve represents the path of a seismic wave travelling through the earth from an earthquake near the surface at A to a monitoring station at B. The radius of the earth is 6·4 units and the path of this wave is a circular arc of radius 29·1 units, where 1 unit = 1000 km. Based on information from other stations, it is known that this particular path just touches the earth's core. The angle AOB measures 104°, where O is the centre of the earth.

Find the radius of the earth's core.
(There is space for work on the next page.)

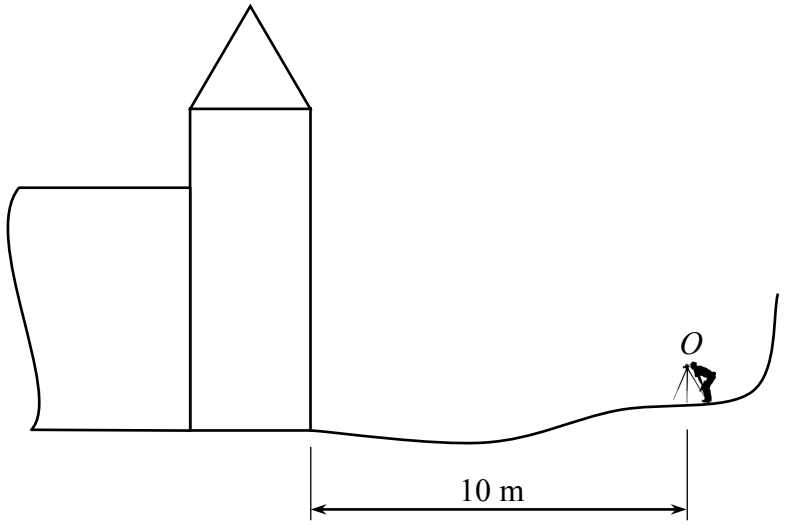


Question 8 **(75 marks)**

(a) A tower that is part of a hotel has a square base of side 4 metres and a roof in the form of a pyramid. The owners plan to cover the roof with copper. To find the amount of copper needed, they need to know the total area of the roof.

A surveyor stands 10 metres from the tower, measured horizontally, and makes observations of angles of elevation from the point *O* as follows:

The angle of elevation of the top of the roof is 46°.
The angle of elevation of the closest point at the bottom of the roof is 42°.
The angle of depression of the closest point at the bottom of the tower is 9°.



(i) Find the vertical height of the roof.

(ii) Find the total area of the roof.

(iii) If all of the angles observed are subject to a possible error of ±1°, find the range of possible areas for the roof.

(b) Twenty five students each measure and record a particular angle of elevation, in degrees, each using his or her own home-made clinometer. The results are as follows:

24	20	22	15	70
15	16	15	16	15
18	16	21	21	73
16	20	12	18	20
18	18	14	22	18

(i) Find what you consider to be the best estimate of the true value of the angle, explaining your reasoning.

(ii) Based on previous experience, a teacher has claimed that, in these circumstances, half of all students will measure the angle correctly to within two degrees. Taking these students to be a simple random sample, and assuming the true value of the angle is the one you calculated in part **(i)**, is there sufficient evidence to reject the teacher's claim at the 5% level of significance?

You may use this page for extra work.

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