



Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Leaving Certificate Examination 2016

# Mathematics

Paper 2

Higher Level

Monday 13 June      Morning 9:30 – 12:00

300 marks

Examination number
--------------------

Centre stamp
--------------

Running total	
---------------	--

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade
-------

## Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if you do not show all necessary work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

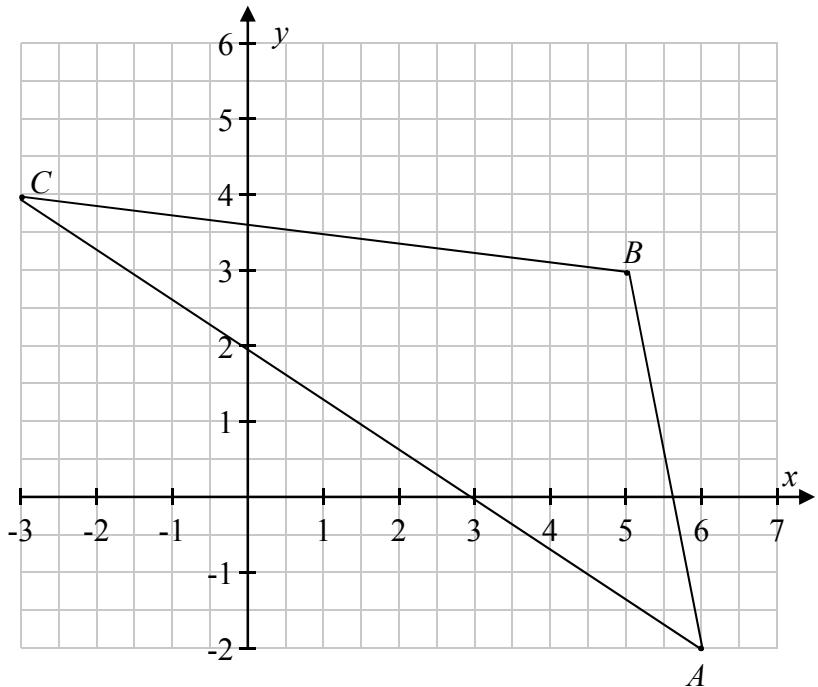
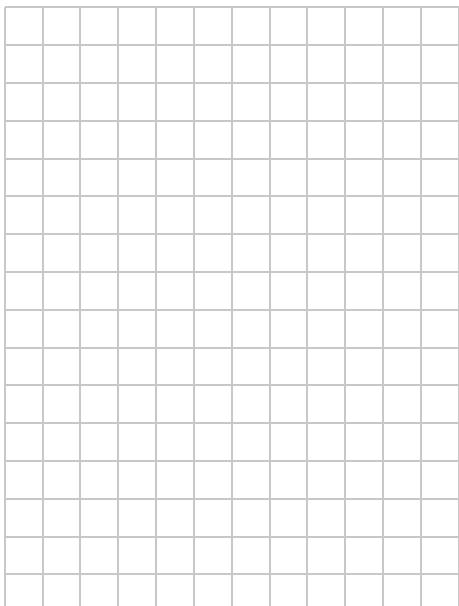
Answer **all six** questions from this section.

**Question 1**

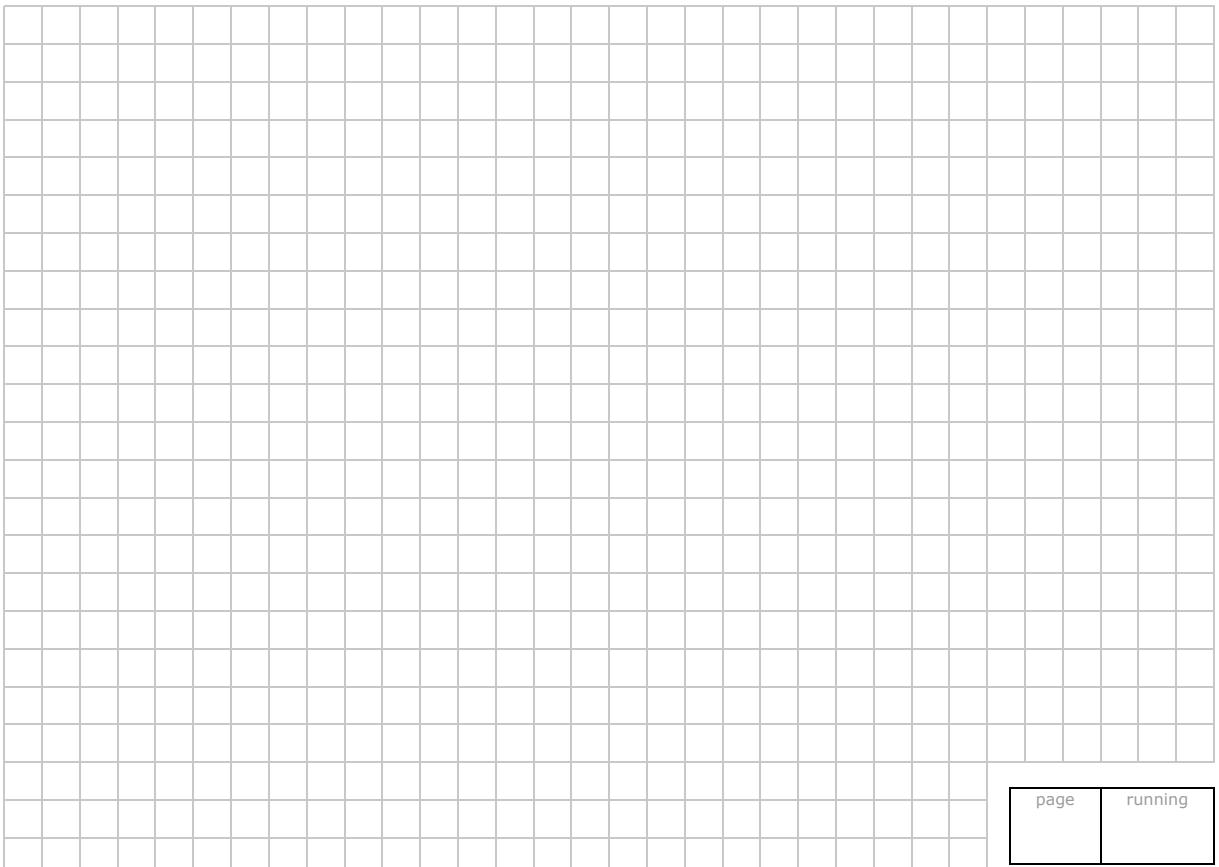
(25 marks)

The points  $A(6, -2)$ ,  $B(5, 3)$  and  $C(-3, 4)$  are shown on the diagram.

- (a) Find the equation of the line through  $B$  which is perpendicular to  $AC$ .



- (b) Use your answer to part (a) above to find the co-ordinates of the orthocentre of the triangle  $ABC$ .

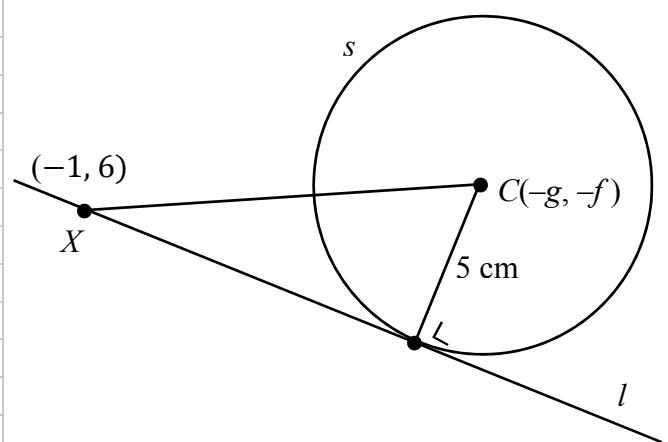


page	running
------	---------

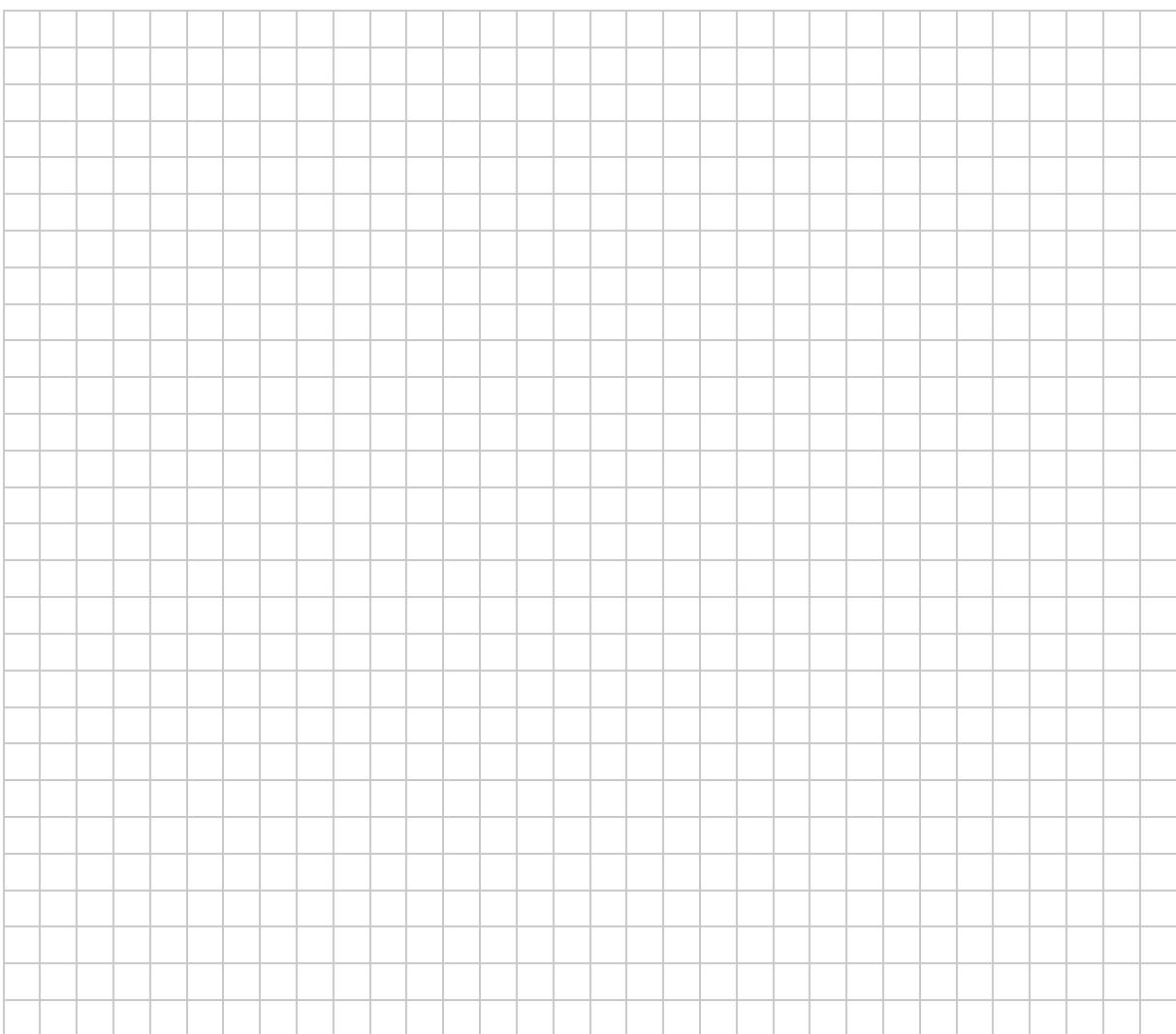
**Question 2****(25 marks)**

A point  $X$  has co-ordinates  $(-1, 6)$  and the slope of the line  $XC$  is  $\frac{1}{7}$ .

- (a) Find the equation of  $XC$ . Give your answer in the form  $ax + by + c = 0$ , where  $a, b, c \in \mathbb{Z}$ .



- (b)  $C$  is the centre of a circle  $s$ , of radius 5 cm. The line  $l: 3x + 4y - 21 = 0$  is a tangent to  $s$  and passes through  $X$ , as shown. Find the equation of one such circle  $s$ .



**Question 3****(25 marks)**

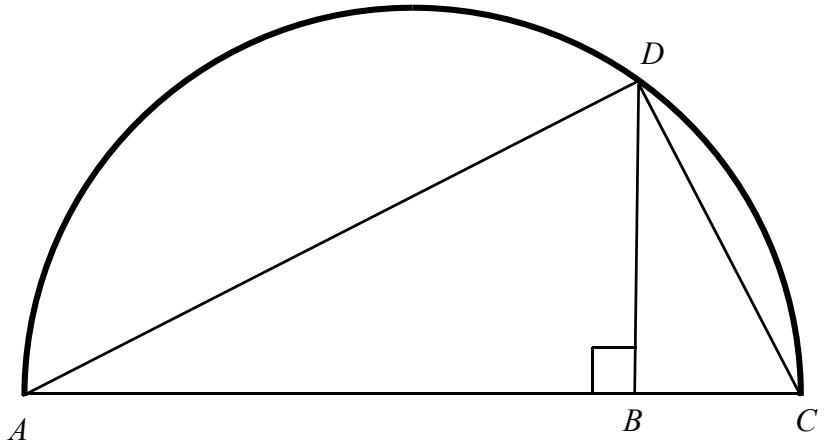
- (a) Show that  $\frac{\cos 7A + \cos A}{\sin 7A - \sin A} = \cot 3A$ .

- (b) Given that  $\cos 2\theta = \frac{1}{9}$ , find  $\cos \theta$  in the form  $\pm \frac{\sqrt{a}}{b}$ , where  $a, b \in \mathbb{N}$ .

**Question 4****(25 marks)**

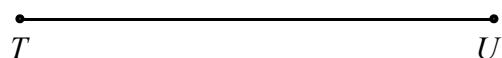
The diagram shows a semi-circle standing on a diameter  $[AC]$ , and  $[BD] \perp [AC]$ .

- (a) (i) Prove that the triangles  $ABD$  and  $DBC$  are similar.



- (ii) If  $|AB| = x$ ,  $|BC| = 1$ , and  $|BD| = y$ , write  $y$  in terms of  $x$ .

- (b)** Use your result from part **(a)(ii)** to **construct** a line segment equal in length (in centimetres) to the square root of the length of the line segment  $[TU]$  which is drawn below.



Previous	page	running
----------	------	---------

**Question 5****(25 marks)**

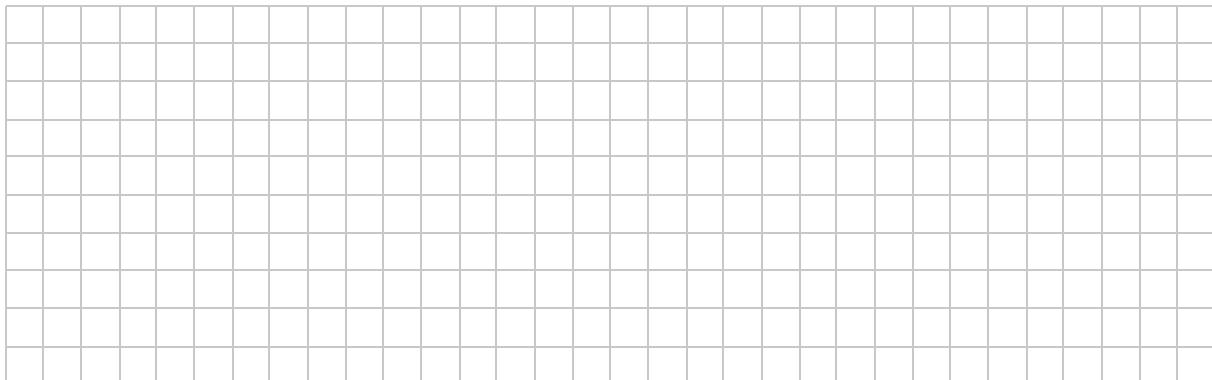
- (a) (i) In an archery competition, the team consisting of John, David, and Mike will win 1<sup>st</sup> prize if at least two of them hit the bullseye with their last arrows. From past experience, they know that the probability that John, David, and Mike will hit the bullseye on their last arrow is  $\frac{1}{5}$ ,  $\frac{1}{6}$ , and  $\frac{1}{4}$  respectively.

Complete the table below to show all the ways in which they could win 1<sup>st</sup> prize.

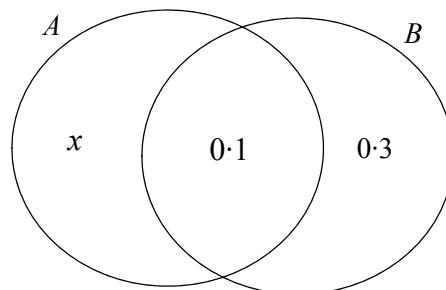
	<b>Way 1</b>	<b>Way 2</b>	<b>Way 3</b>	<b>Way 4</b>
John	✓			
David	✓			
Mike	✗			

✓ = Hit  
✗ = Miss

- (ii) Hence or otherwise find the probability that they will win the competition.



- (b) Two events,  $A$  and  $B$ , are represented in the diagram.  
 $P(A \cap B) = 0.1$ ,  $P(B \setminus A) = 0.3$  and  $P(A \setminus B) = x$ .  
Write  $P(A)$  in terms of  $x$  and hence, or otherwise, find the value of  $x$  for which the events  $A$  and  $B$  are independent.



$P(A) =$

$x =$



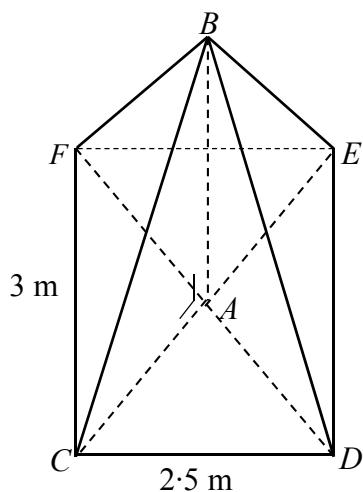
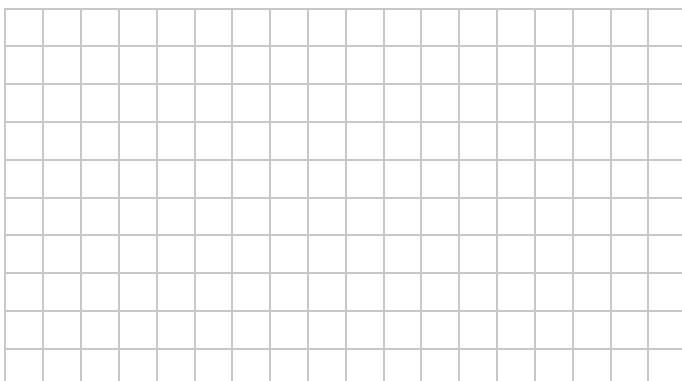
Answer **all three** questions from this section.

**Question 7****(55 marks)**

A glass Roof Lantern in the shape of a pyramid has a rectangular base  $CDEF$  and its apex is at  $B$  as shown. The vertical height of the pyramid is  $|AB|$ , where  $A$  is the point of intersection of the diagonals of the base as shown in the diagram.

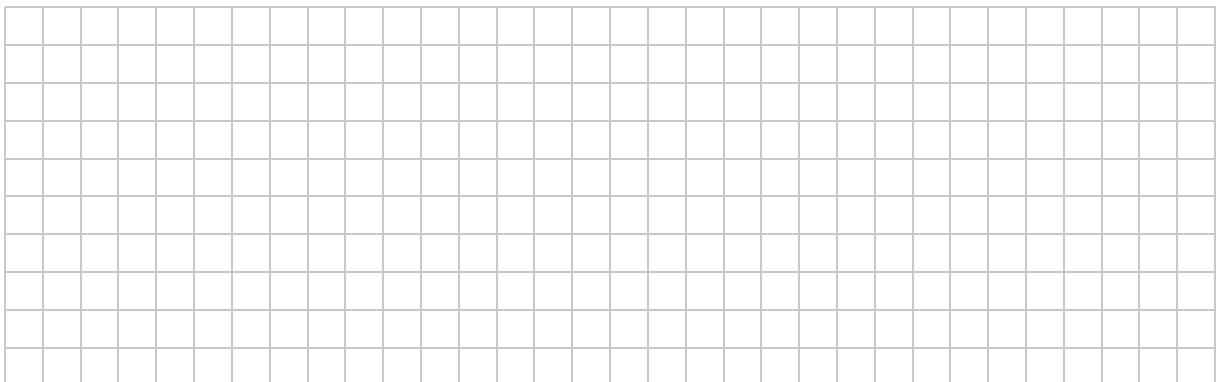
Also  $|CD| = 2.5$  m and  $|CF| = 3$  m.

- (a) (i) Show that  $|AC| = 1.95$  m, correct to two decimal places.

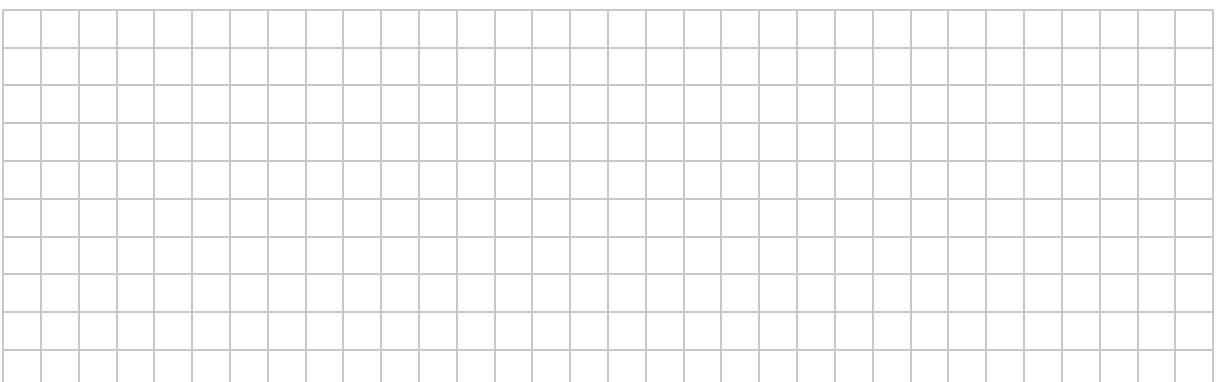


- (ii) The angle of elevation of  $B$  from  $C$  is  $50^\circ$  (i.e.  $|\angle BCA| = 50^\circ$ ).

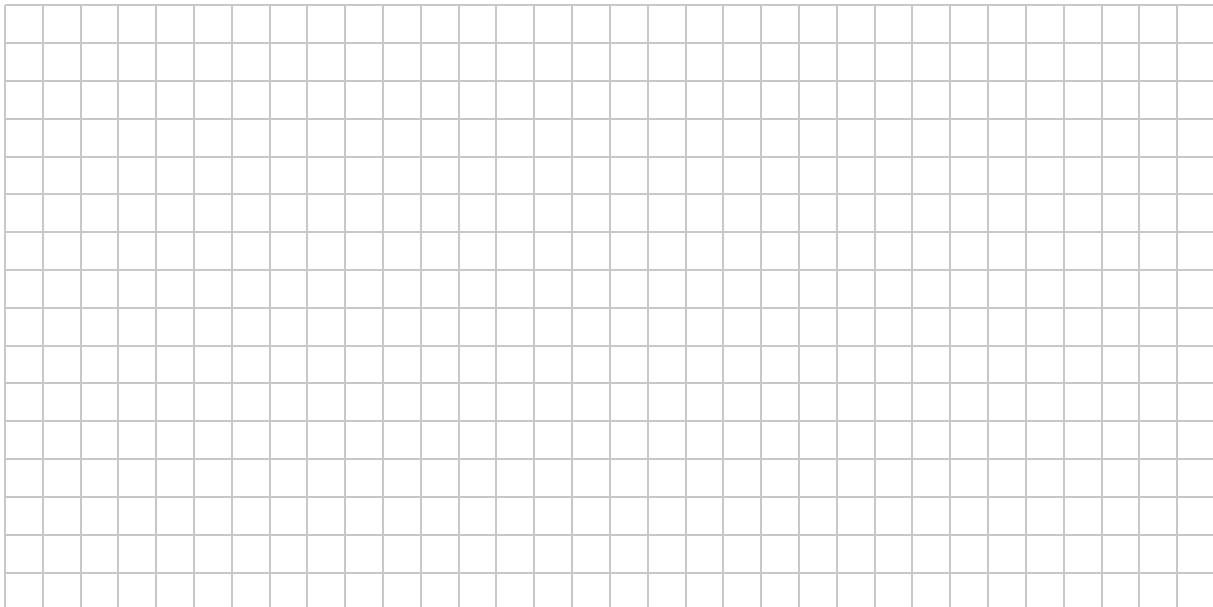
Show that  $|AB| = 2.3$  m, correct to one decimal place.



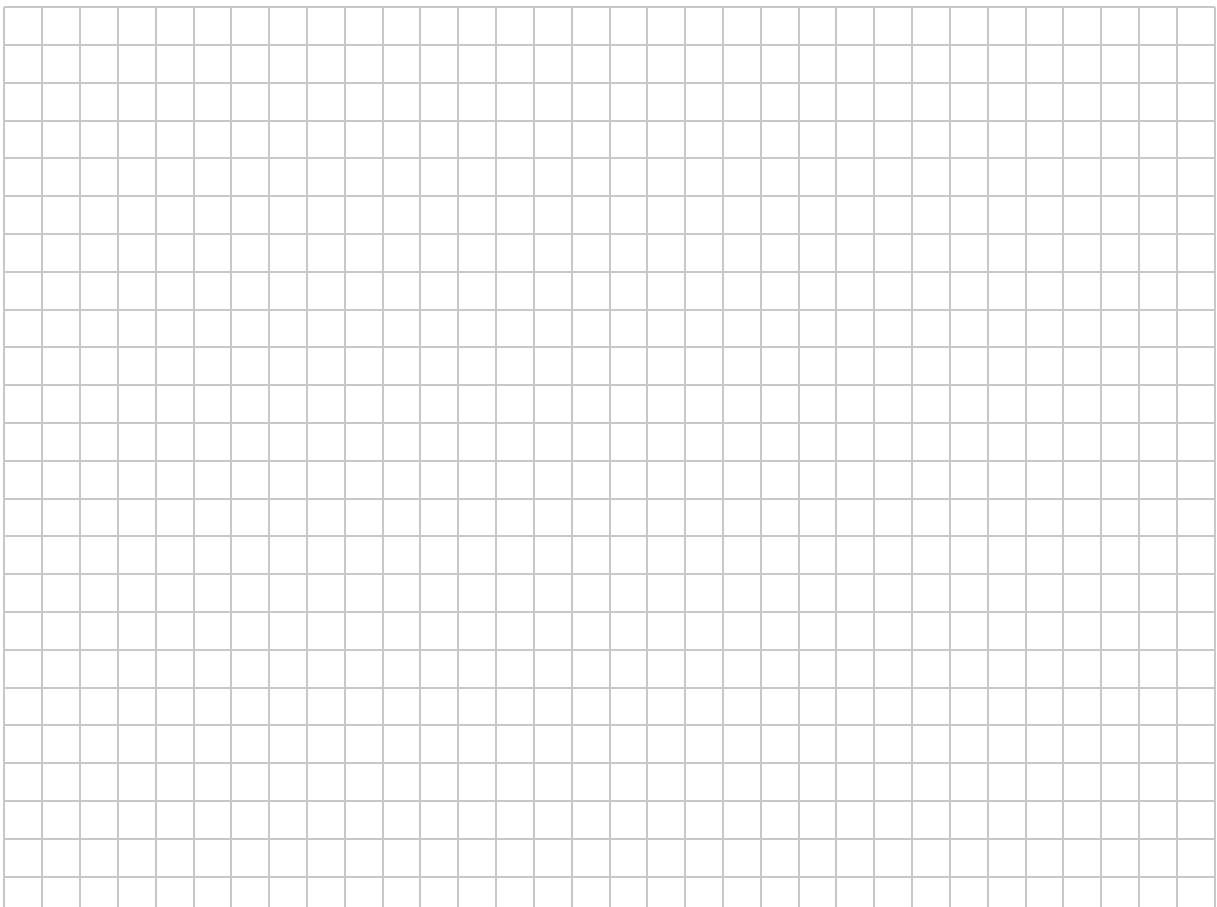
- (iii) Find  $|BC|$ , correct to the nearest metre.



**(iv)** Find  $|\angle BCD|$ , correct to the nearest degree.



**(v)** Find the area of glass required to glaze all four triangular sides of the pyramid.  
Give your answer correct to the nearest  $\text{m}^2$ .



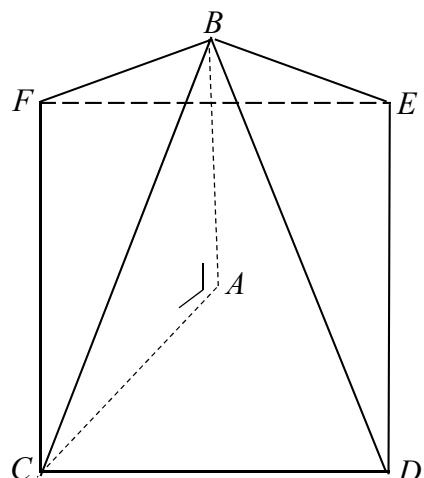
- (b) Another Roof Lantern, in the shape of a pyramid, has a square base  $CDEF$ . The vertical height  $|AB| = 3$  m, where  $A$  is the point of intersection of the diagonals of the base as shown.

The angle of elevation of  $B$  from  $C$  is  $60^\circ$

(i.e.  $|\angle BCA| = 60^\circ$ ).

Find the length of the side of the square base of the lantern.

Give your answer in the form  $\sqrt{a}$  m, where  $a \in \mathbb{N}$ .



**Question 8****(45 marks)**

The height of the water in a port was measured over a period of time. The average height was found to be 1·6 m. The height measured in metres,  $h(t)$ , was modelled using the function

$$h(t) = 1\cdot6 + 1\cdot5 \cos\left(\frac{\pi}{6}t\right)$$

where  $t$  represents the number of hours since the last recorded high tide and  $\left(\frac{\pi}{6}t\right)$  is expressed in radians.

- (a)** Find the period and range of  $h(t)$ .

Period:

Range:

- (b)** Find the maximum height of the water in the port.

- (c)** Find the rate at which the height of the water is changing when  $t = 2$ , correct to two decimal places. Explain your answer in the context of the question.

Rate:

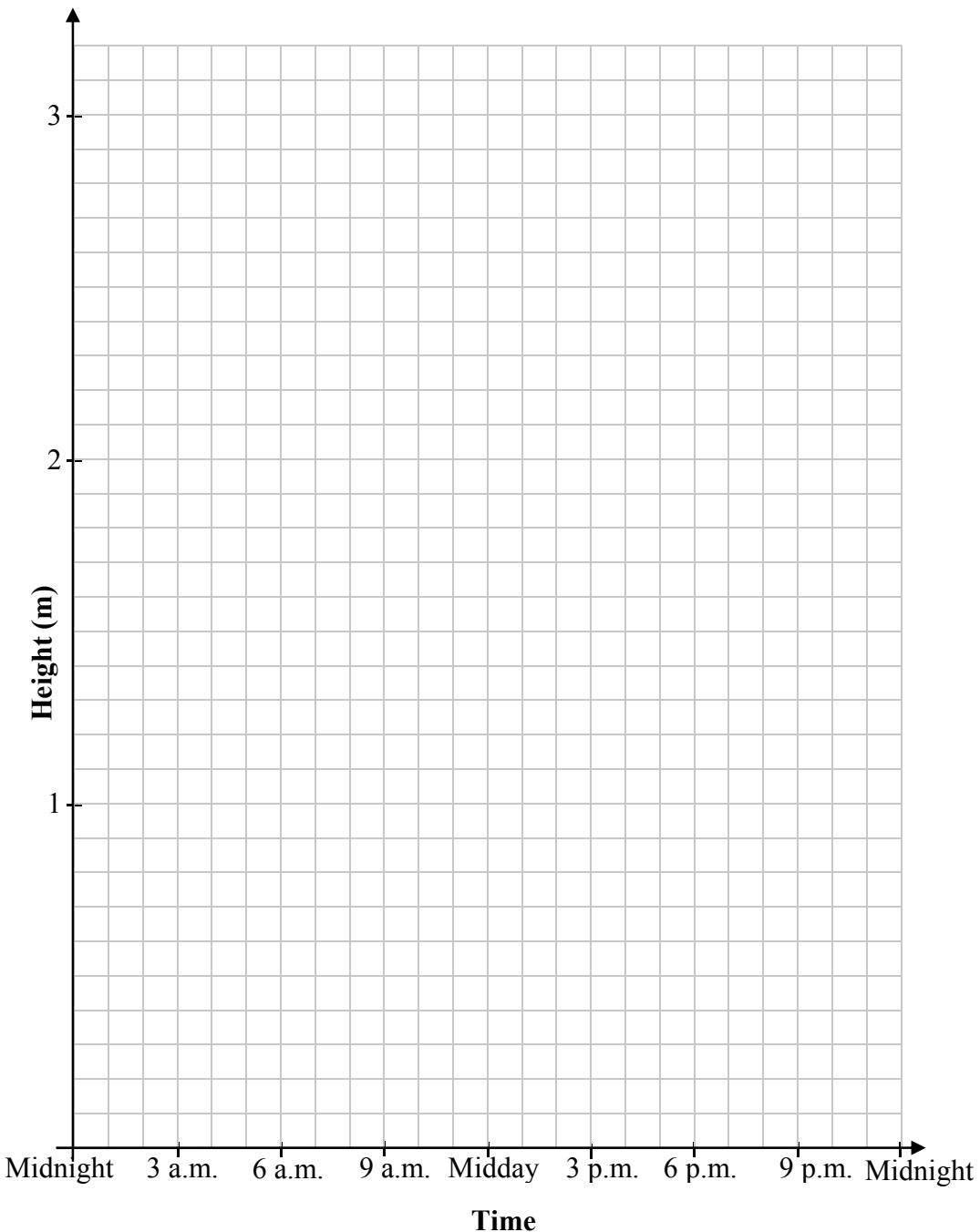
Explanation:

Previous	page	running
----------	------	---------

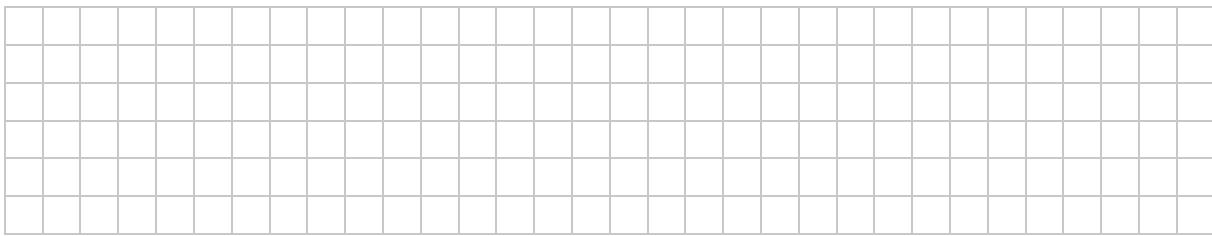
- (d) (i) On a particular day the high tide occurred at midnight (i.e.  $t = 0$ ). Use the function to complete the table and show the height,  $h(t)$ , of the water between midnight and the following midnight.

$h(t) = 1.6 + 1.5 \cos\left(\frac{\pi}{6}t\right)$									
Time	Midnight	3 a.m.	6 a.m.	9 a.m.	12 noon	3 p.m.	6 p.m.	9 p.m.	Midnight
$t$ (hours)	0	3							
$h(t)$ (m)									

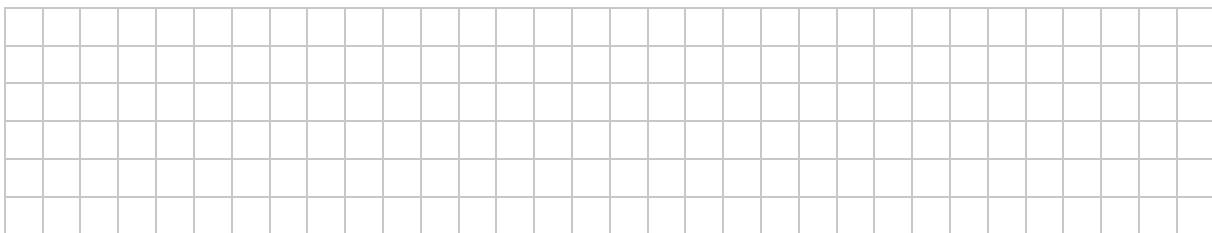
- (ii) Sketch the graph of  $h(t)$  between midnight and the following midnight.



- (e) Find, from your sketch, the difference in water height between low tide and high tide.



- (f) A fully loaded barge enters the port, unloads its cargo and departs some time later.  
The fully loaded barge requires a minimum water level of 2 m.  
When the barge is unloaded it only requires 1.5 m.  
Use your graph to estimate the **maximum** amount of time that the barge can spend in port, without resting on the sea-bed.

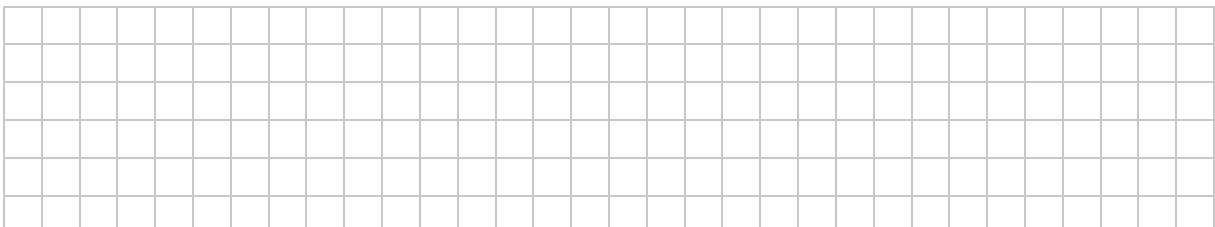


**Question 9****(50 marks)**

Data on earnings were published for a particular country. The data showed that the annual income of people in full-time employment was normally distributed with a mean of €39 400 and a standard deviation of €12 920.

- (a) (i) The government intends to impose a new tax on incomes over €60 000.

Find the percentage of full-time workers who will be liable for this tax, correct to one decimal place.



- (ii) The government will also provide a subsidy to the lowest 10 % of income earners.

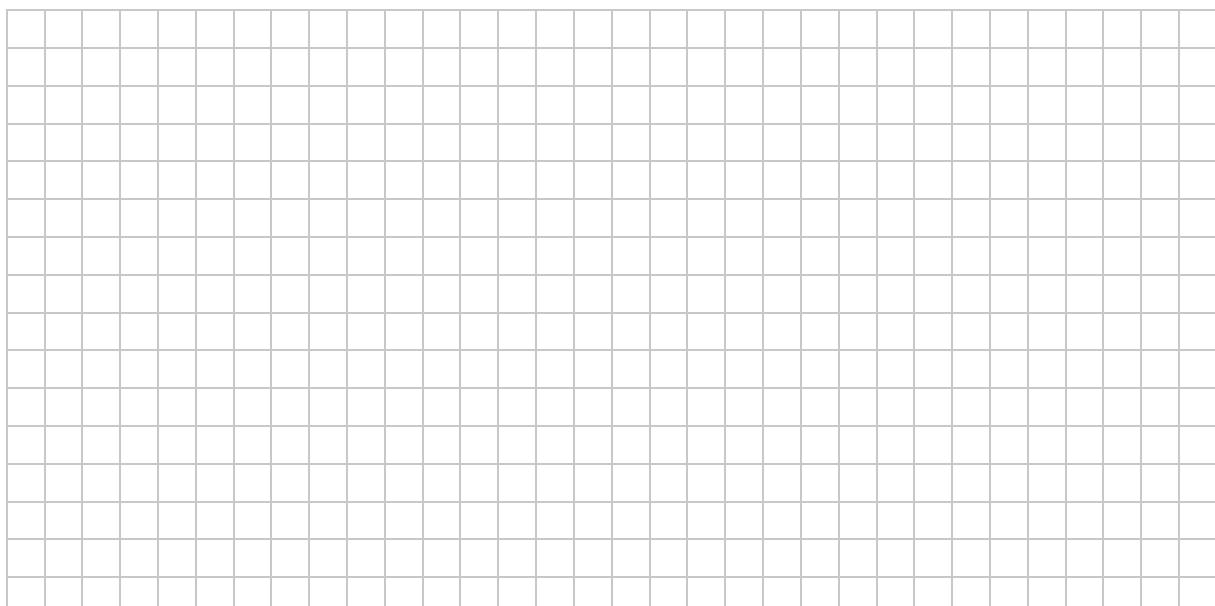
Find the level of income at which the government will stop paying the subsidy, correct to the nearest euro.



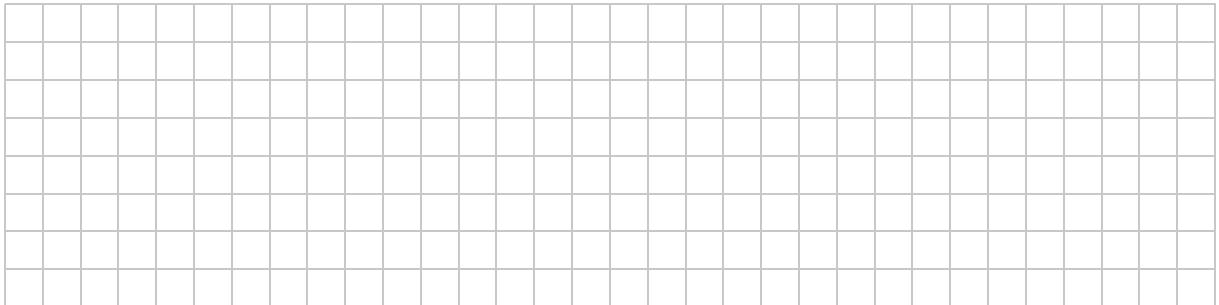
- (iii) Some time later a research institute surveyed a sample of 1000 full-time workers, randomly selected, and found that the mean annual income of the sample was €38 280. Test the hypothesis, at the 5 % level of significance, that the mean annual income of full-time workers has changed since the national data were published.

State the null hypothesis and the alternative hypothesis.

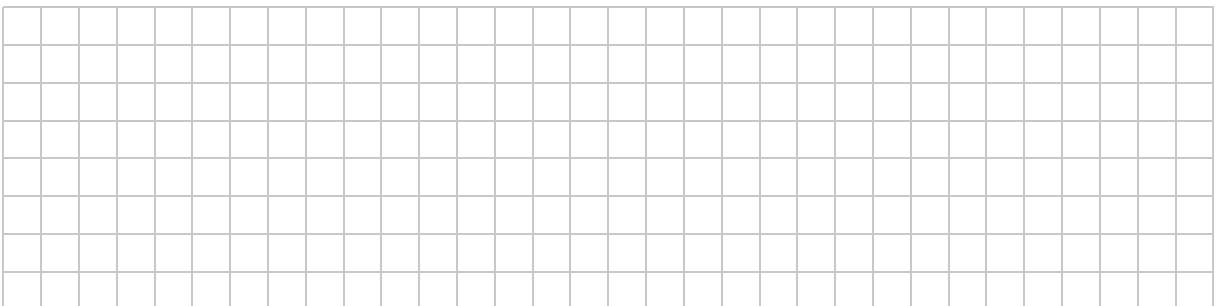
Give your conclusion in the context of the question.



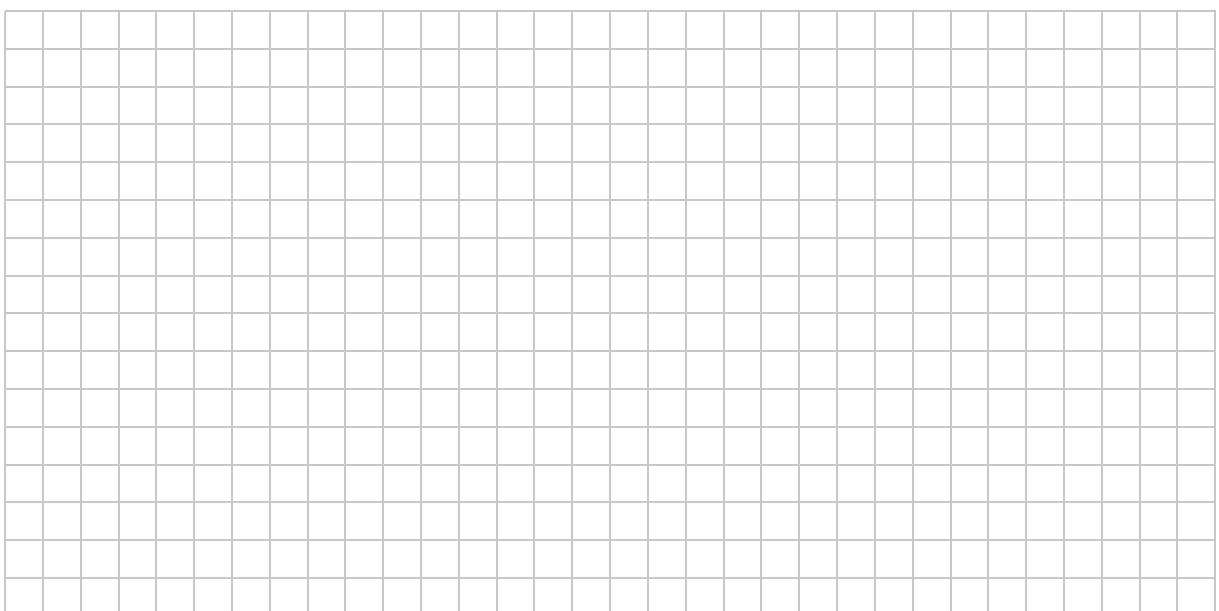
- (b) The research institute surveyed 400 full-time farmers, randomly selected from all the full-time farmers in the country, and found that the mean income for the sample was €26 974 and the standard deviation was €5120.  
Assuming that annual farm income is normally distributed in this country, create a 95 % confidence interval for the mean income of full-time farmers.



- (c) It is known that data on farm size are not normally distributed.  
The research institute could take many large random samples of farm size and create a sampling distribution of the means of all these samples.  
Give one reason why they might do this.

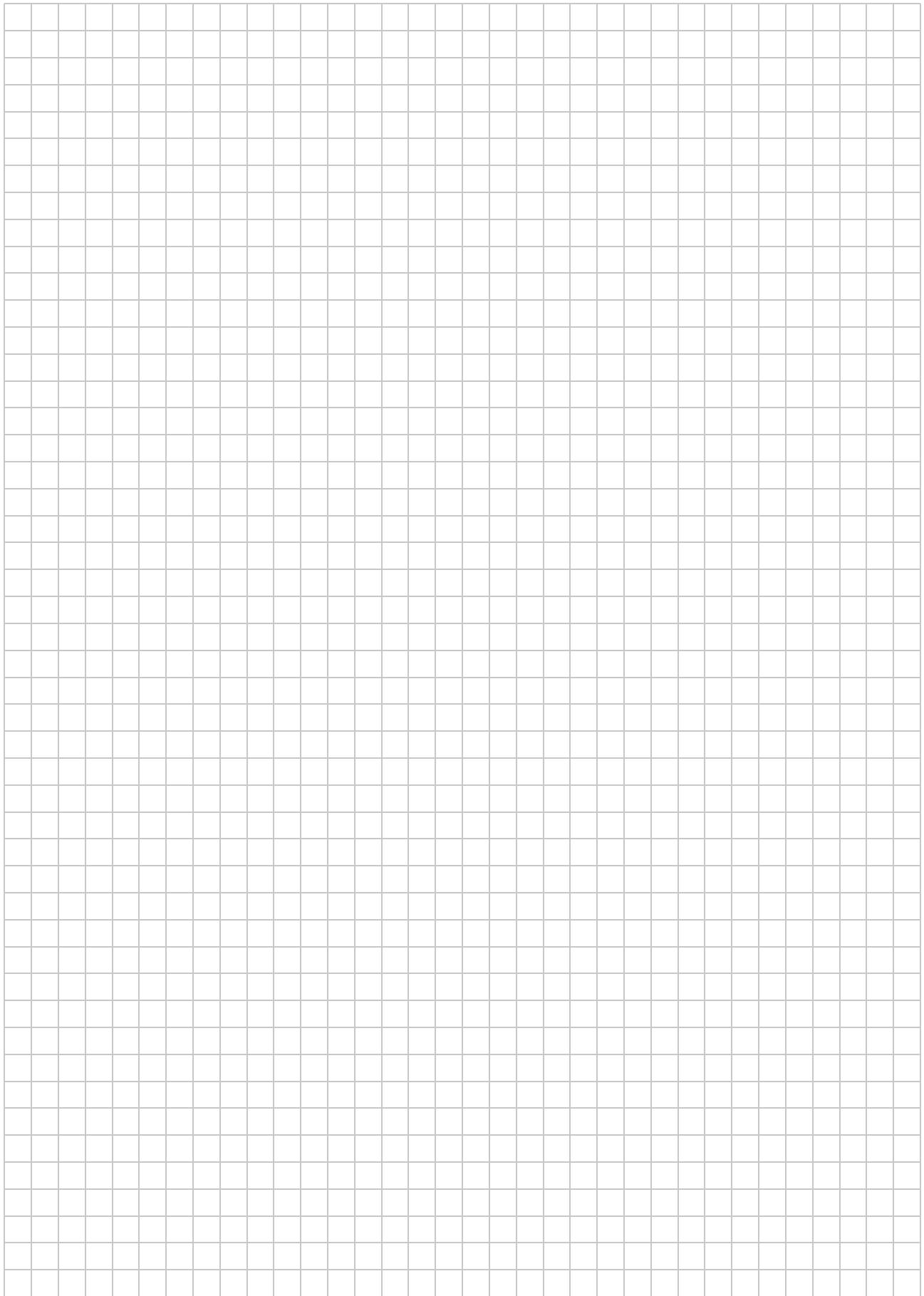


- (d) The research institute also carried out a survey into the use of agricultural land.  
 $n$  farmers were surveyed.  
If the margin of error of the survey was 4·5 %, find the value of  $n$ .

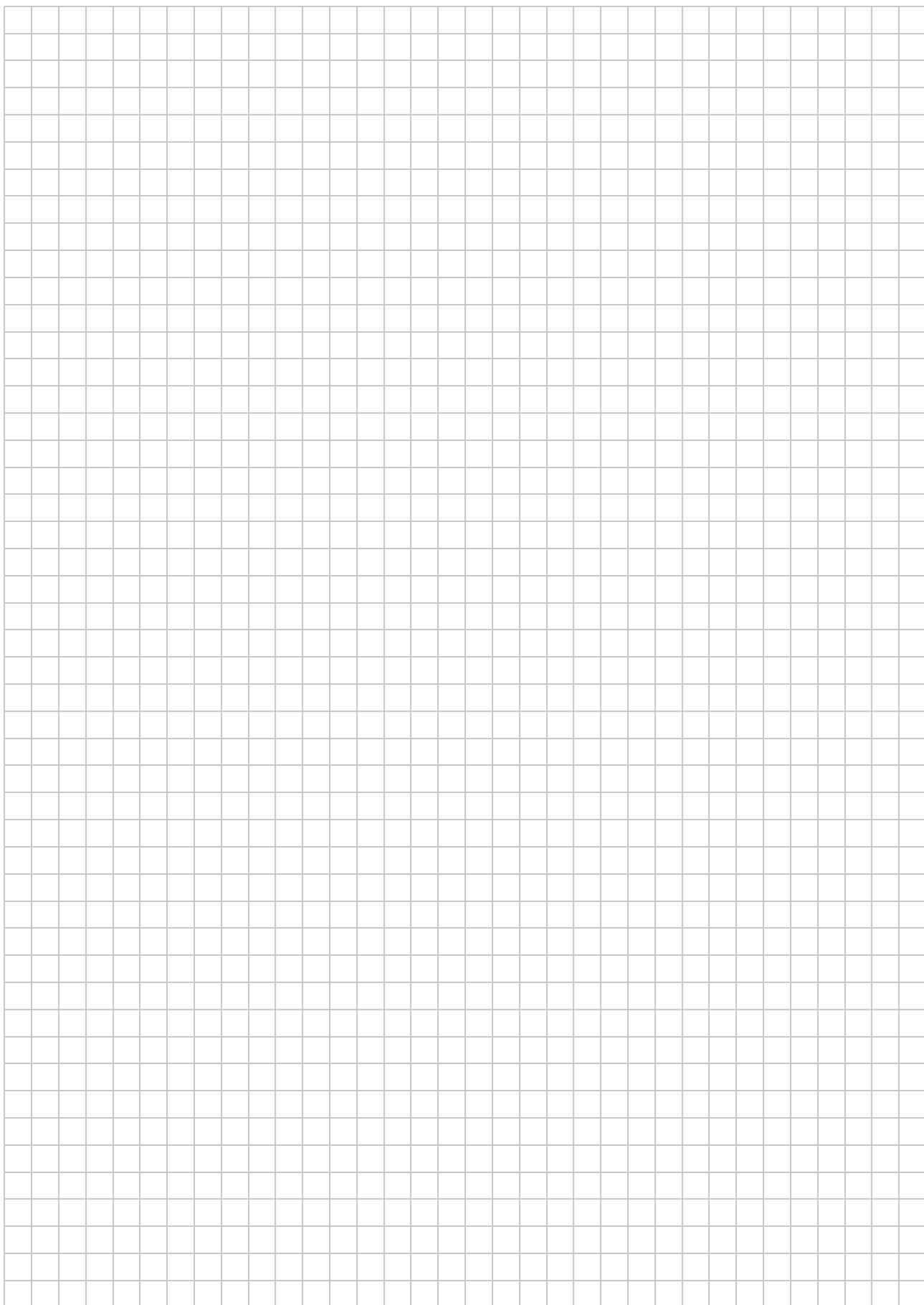


Previous	page	running
----------	------	---------

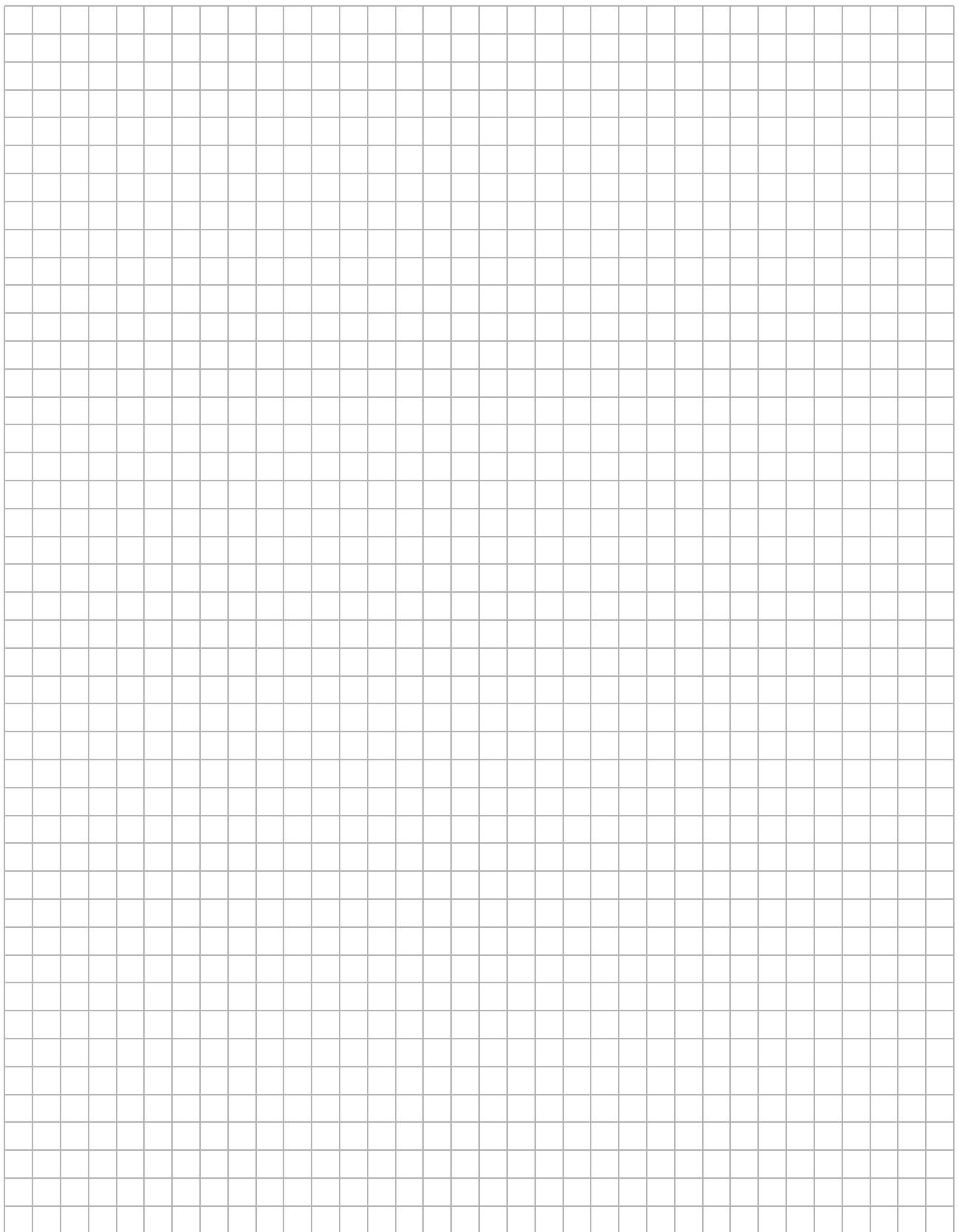
You may use this page for extra work.



You may use this page for extra work.



Previous	page	running
----------	------	---------



Leaving Certificate 2016 – Higher Level

## **Mathematics – Paper 2**

Monday 13 June

Morning 9:30 – 12:00