Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2018

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

Contents

Page

Paper 1

Paper 2

Blank Page

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2018

Mathematics

Higher Level

Paper 1

Solutions and Marking scheme

300 marks

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales		0, 2, 5	0, 3, 4, 5		
10 mark scales			0, 4, 8, 10	0, 3, 5, 8, 10	
15 mark scales			0, 5, 10, 15	0, 5, 7, 11, 15	
20 mark scales				0, 5, 10, 15, 20	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

Marking Scheme

Section A

Question 1	
(a)	15D
(b)	10D

Question 2	
(a)	10C
(b)	10C
(c)	5C

Question 3	
(a)	10D
(b)	15D

Question 4	
(a)	15D
(b)	10C

Question 5	
(a)(i)	10C
(a)(ii)	10D
(b)	5C

Question 6	
(a)	10C
(b)(i)	10C
(b)(ii)	5B

Section B

Question 7	
(a)	15C
(b)	5C
(c)	20D
(d)	5C
(e)(i)	5B
(e)(ii)	5C

Question 8	
(a)	10C
(b)	10C
(c)	10C
(d)	10D

Question 9	
(a)	10C
(b)(i)	5B
(b)(ii)	5C
(c)(i)	10C
(c)(ii)	5B
(d)(i)	10C
(d)(ii)	5C
(d)(iii)	5C

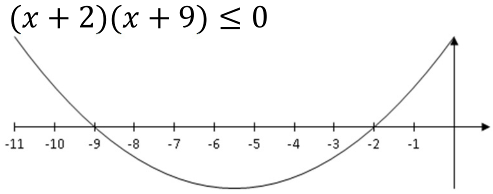
Note: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.

Throughout the scheme indicate by use of * where an arithmetic error occurs.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$(i)\quad 2x+3y-z=-4 \qquad \times(2)$ $(ii)\quad 3x+2y+2z=14 \qquad \times(-3)$ $4x+6y-2z=-8$ $-9x-6y-6z=-42$ $-5x-8z=-50$ $(iii)\quad x-3z=-13 \qquad \times\ (5)$ $-5x-8z=-50$ $5x-15z=-65$ $-23z=-115$ $z=5$ $\Rightarrow x=2$ $\Rightarrow y=-1 \qquad \{2,-1,5\}$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* Matches coefficient of 1 variable in 2 equations Writes x in terms of z in eq (*iii*) *Mid Partial Credit:* 1 unknown found with errors Eliminates one unknown 1 unknown found and stops *High Partial Credit:* 2 unknowns found
(b)	$\dfrac{2x-3}{x+2}\geq 3 \qquad \times\ (x+2)^2$ $(2x-3)(x+2)\geq 3(x+2)^2$ $2x^2+x-6\geq 3x^2+12x+12$ $x^2+11x+18\leq 0$ $(x+2)(x+9)\leq 0$  $-9\leq x<-2$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit* Use of $(x+2)^2$ Relevant work but with linear inequality Squares both sides with some subsequent work (low partial credit at most) *Mid Partial Credit:* Quadratic inequality involving 0 *High Partial Credit* Roots of quadratic found **Note**: Accept $-9\leq x\leq -2$

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$\dfrac{5x-8}{x^2}=\dfrac{x+8}{5x-8}$ $(5x-8)^2=x^2(x+8)$ $25x^2-80x+64=x^3+8x^2$ $x^3-17x^2+80x-64=0$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $\frac{5x-8}{x^2}$ or $\frac{x+8}{5x-8}$ Some effort at finding r in a geometric sequence (must use at least one of the terms) $r=\frac{T_n}{T_{n-1}}$ or similar *High Partial Credit:* $\dfrac{5x-8}{x^2}=\dfrac{x+8}{5x-8}$ $(5x-8)^2$ and $x^2(x+8)$ *0 credit*: Treats as an arithmetic sequence
(b)	$f(x)=x^3-17x^2+80x-64$ $f(1)=(1)^3-17(1)^2+80(1)-64=0$ $\Rightarrow (x-1)$ is a factor $x^3-17x^2+80x-64=0$ $x^2(x-1)-16x(x-1)+64(x-1)$ $x^2-16x+64=0$ $(x-8)(x-8)=0$ $x=8$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Shows $f(1)=0$ Any correct substitution *High Partial Credit:* Quotient in quadratic form found Accept $x=8$ without work if $f(1)=0$ has been shown

(c)	$\underline{x = 1}$ $1^2, \quad 5(1) - 8, \quad 1 + 8$ $1, -3, 9$ which doesn't have a sum to infinity $(\lvert r \rvert > 1)$ $\underline{x = 8}$ $8^2, \quad 5(8) - 8, \quad 8 + 8$ $64, 32, 16 \ldots.$ $a = 64$ and $r = \frac{1}{2}$ $S_\infty = \frac{a}{1-r} = \frac{64}{1-\frac{1}{2}} = \frac{64}{\frac{1}{2}} = 128$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Substitution used to identify $x = 8$ as the required value Substitution used to exclude $x = 1$ as the required value Finds $\frac{a}{1-r}$ for $x = 1$ $S_\infty = \dfrac{x^2}{1 - \dfrac{5x-8}{x^2}}$ Relevant substitution into correct formula *High Partial Credit:* GP identified (*a* and *r*) If the candidate works with both $x = 1$ **and** $x = 8$ but fails to eliminate $x = 1$ or chooses the incorrect answer **Note**: if $\lvert r \rvert > 1$ then Low Partial Credit at most

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$h'(x) = -2\sin(2x)$ At $x = \frac{\pi}{3}$: $h'\left(\frac{\pi}{3}\right) = -2\sin\left(\frac{2\pi}{3}\right)$ $= -2\left(\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$ $\tan\theta = -\sqrt{3}$ $\theta = 120°$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Differentiation indicated Use of 2 *Mid Partial Credit:* Derivative found *High Partial Credit:* $\tan\theta =$ evaluated derivative $\theta = -60°$ **Note**: Must use differentiation to gain any credit **Note**: If integration symbol appears then 0 credit
(b)	$\frac{1}{\frac{\pi}{4} - 0}\int_0^{\frac{\pi}{4}} \cos(2x)\, dx$ $= \frac{4}{\pi}\left(\frac{\sin(2x)}{2}\right)_0^{\frac{\pi}{4}}$ $= \frac{4}{\pi}\left(\frac{\sin\frac{\pi}{2}}{2} - \frac{\sin 0}{2}\right)$ $= \frac{4}{\pi}\left(\frac{1}{2}\right) = \frac{2}{\pi}$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* Integration indicated *Mid Partial Credit:* $\cos 2x$ integrated correctly $\left(\frac{\sin(2x)}{2}\right)$ $-2\sin 2x$ and finishes correctly *High Partial Credit:* Substitutes limits into integral and stops Integral evaluated at $x = \frac{\pi}{4}$ (i.e. omits $\frac{1}{\frac{\pi}{4}-0}$) and finishes **Note**: errors in integration could include An error in the trig function (including sign) An error in the angle An error in the application of the chain rule **Note**: Must have integration to gain any credit

<table>
<tr><th>Q4</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

P(1)
$(\cos\theta + i\sin\theta)^1 = \cos(1\theta) + i\sin(1\theta)$

$\boldsymbol{P(k)}$: Assume $(\cos\theta + i\text{sin}\theta)^k = \cos(k\theta) + i\text{sin}(k\theta)$

Test $\boldsymbol{P(k+1)}$:
$(\cos\theta + i\text{sin}\theta)^{k+1} =$
$= \cos(k+1)\theta + i\text{sin}(k+1)\theta$

$(\cos\theta + i\text{sin}\theta)^{k+1}$
$= (\cos\theta + i\text{sin}\theta)^k.(\cos\theta + i\text{sin}\theta)^1$

$= (\cos(k\theta) + i\text{sin}(k\theta)).(\cos\theta + i\text{sin}\theta)$

$= [\cos(k\theta)\cos\theta - \sin(k\theta)\text{sin}\theta]$
$+ i[\cos(k\theta)\sin\theta + \cos\theta\sin(k\theta)]$

$= \cos(k+1)\,\theta + isin(k+1)\theta$

Thus the proposition is true for $n = k+1$ provided it is true for $n = k$ but it is true for $n = 1$ and therefore true for all positive integers.

</td><td>

Scale 15D (0, 5, 7, 11, 15)
Low Partial Credit:
Step $P(1)$

Mid Partial Credit:
Step $P(k)$ or Step $P(k+1)$

High Partial Credit:
Uses Step $P(k)$ to prove Step $P(k+1)$

Note: Accept Step $P(1)$, Step $P(k)$, Step $P(k+1)$ in any order

Full credit –1:
Omits conclusion but otherwise correct

Full credit:
$[r(\cos\theta + i\text{sin}\theta)]^n = r^n\,(\cos(n\theta) + i\text{sin}(n\theta))$
proved correctly

</td></tr>
<tr><td>(b)</td><td>

$\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3 = 1\left(\cos\frac{2\pi}{3} + i\text{sin}\frac{2\pi}{3}\right)^3$
$= \left(\cos(3)\frac{2\pi}{3} + i\sin(3)\frac{2\pi}{3}\right)$
$= (\cos 2\pi + i\text{sin}2\pi) =$
$1 + 0i$
$= 1$

</td><td>

Scale 10C (0, 4, 8, 10)
Low Partial Credit:
Modulus or argument correct
Some correct multiplication
Apply De Moivre correctly with incorrect modulus and argument

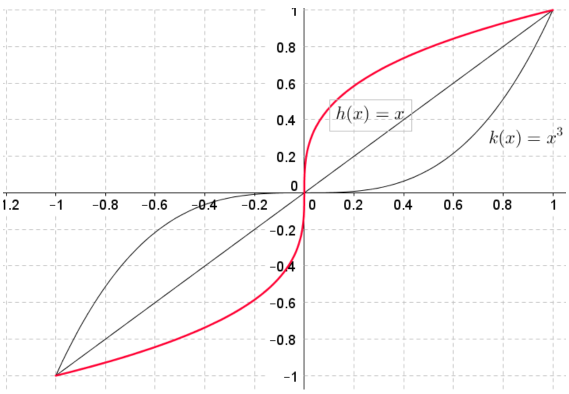
High Partial Credit:
$\left(\cos(3)\frac{2\pi}{3} + i\sin(3)\frac{2\pi}{3}\right)$
Multiplication correct but un-simplified

Full credit –1:
$\cos 2\pi + i\text{sin}2\pi$ or $\cos 360° + i\text{sin}360°$

Accept: Answer with reference to cube root of unity

</td></tr>
</table>

Q5	Model Solution – 25 Marks	Marking Notes
(a) **(i)**	$row\ 2: S_{45} = \frac{45}{2}[14 + 44(5)] = 5265$ $row\ 1: S_{45} = \frac{45}{2}[8 + 44(3)] = 3150$ $\therefore$ Difference $= 2115$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Formulates S_{45} for row 1 or row 2 3+5+7 … Identifies *a* or *r* for either row 1 or row 2 *High Partial Credit:* S_{45} found for row 1 or row 2 *Full credit* -1: Fails to subtract
(a) **(ii)**	T_1(in row 60): $T_{60} = 4 + (60-1)3 = 181$ T_2 (in row 60) $= T_{60}$ of $7, 12, 17, 22 \ldots.$ $T_{60} = 7 + (60-1)5 = 302$ $\therefore T_{70}$ of $181, 302, \ldots \ldots$ $= 181 + (70-1)121 = 8530$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Identifies T_{60} in column 1 or T_{70} In row 1 or equivalent Some relevant substitution into correct formula Identifies *a* or *d* for either row 1 or row 2 *Mid Partial Credit:* Finds *a* in row 60 or row 70 Finds *d* in row 60 or row 70 *High Partial Credit:* Formulates substituted expression for T_{70} in row 60 or T_{60} in column 70 Finds both *a* and *d* in row 60 or row 70
(b)	$a_3 = a_2 - a_1 = 2 - 4 = -2$ $a_4 = a_3 - a_2 = -2 - 2 = -4$ $a_5 = a_4 - a_3 = -4 - (-2) = -2$ $a_6 = a_5 - a_4 = -2 - (-4) = 2$ $a_7 = a_6 - a_5 = 2 - (-2) = 4$ $a_8 = a_7 - a_6 = 4 - 2 = 2$ Therefore, the sequence consists of a repeating pattern of $4,\ 2, -2, -4, -2,\ 2$ $\therefore a_{2016} = 2$ (multiple of 6) $\Rightarrow a_{2019} = -2$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $a_3 = -2$ $a_3 = a_2 - a_1$ or similar *High Partial Credit:* Any 4 from a_3, a_4, a_5, a_6, a_7 and a_8 found *Full credit* -1: $a_3, a_4, a_5, a_6,$ and a_{2019} found

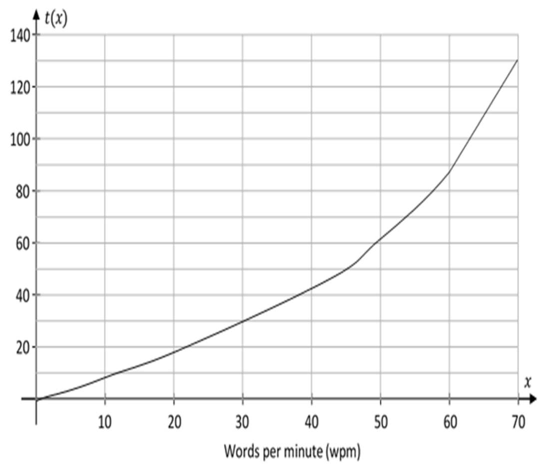
Q6	Model Solution – 25 Marks	Marking Notes
(a)	$x^3 = x$ $\Rightarrow x^3 - x = 0$ $\Rightarrow x(x^2 - 1) = 0$ $x(x-1)(x+1) = 0$ $x = 0$ or $x = \pm 1$ (−1, −1), (0, 0), (1, 1)	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Equation written One correct solution from the graph Solution of the form (a, a) where $a \neq 0, 1$ *High Partial Credit:* Equation factorised (3 factors) 2 correct points x values only
(b) (i)	$2\int_0^1 x - x^3 dx$ $= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]$ $=2\left[\frac{1}{2} - \frac{1}{4} - 0\right] =$ $\frac{1}{2}$ unit^2	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Integral indicated One relevant area found *High Partial Credit:* Integral evaluated at $x = 1$ (upper limit) $\int_{-1}^{1} x - x^3 dx = 0$
(b) (ii)	 	**Scale 5B (0, 2, 5)** *Partial Credit:* Incomplete image 2 correct image points $k^{-1}(x) = x^{\frac{1}{3}}$

Q7	Model Solution – 55 Marks	Marking Notes
(a)	$35{\cdot}96 = k\ln\left(1 - \frac{35}{80}\right)$ $35{\cdot}96 = k\ln\left(\frac{45}{80}\right)$ $k = \frac{35{\cdot}96}{\ln\left(\frac{45}{80}\right)}$ $k = -62{\cdot}5$ to one place of decimals	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Effort at transposing Some substitution into function Full substitution and stops *High Partial Credit:* Function written in terms of k and fully substituted One incorrect substitution worked correctly and with some reference to $k \neq -62{\cdot}5$
(b)	$100 = -62{\cdot}5\ln\left(1 - \frac{x}{80}\right)$ $\frac{100}{-62{\cdot}5} = \ln\left(1 - \frac{x}{80}\right)$ $e^{\frac{100}{-62\cdot5}} = 1 - \frac{x}{80}$ $x = -80(e^{\frac{100}{-62\cdot5}} - 1)$ $x = 64$ wpm (To the nearest whole number)	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Some substitution into function Trial and improvement (more than 1 iteration) Correct answer without work *High Partial Credit:* $e^{\frac{100}{-62\cdot5}} = 1 - \frac{x}{80}$ Equation rewritten in terms of x or $\frac{x}{80} =$

(c)

x (wpm)	0	10	20	30	40	50	60	70
$t(x)$ (days)	0	8	18	29	43	61	87	130

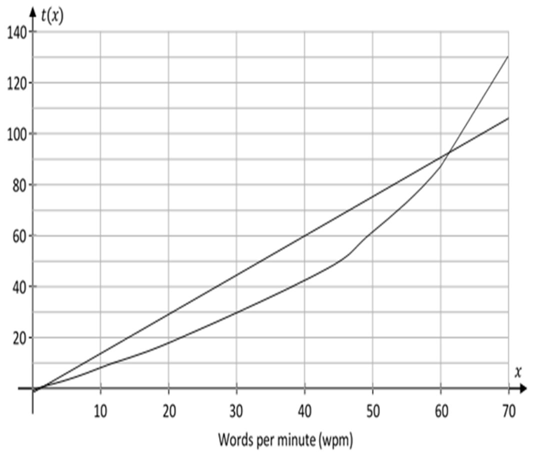
(c)



Scale 20D (0, 5, 10, 15, 20)
Low Partial Credit:
One entry correct
One plot (from candidates table) correct

Mid Partial Credit:
4 entries correct and 4 plots of table values

High Partial Credit:
All plots consistent with candidates table values (with at least 1 correct value)
Table correct but incorrect plots

(d)		**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* One point on line identified One point (not origin) plotted *High Partial Credit:* 2 points on line identified and plotted
(e) (i)	Approx 62 wpm	**Scale 5B(0, 2, 5)** *Partial Credit:* Point of intersection indicated on graph $h(x)$ written in terms of x **Tolerance**: $\pm$ 2 wpm
(e) (ii)	For Maximum Value: Set $h'(x) = 0$ $h(x) = 1{\cdot}5x + 62{\cdot}5\ln\left(1 - \frac{x}{80}\right)$ $h'(x) = 1{\cdot}5 + 62{\cdot}5\left(\frac{1}{1-\frac{x}{80}}\right) \times \left(-\frac{1}{80}\right)$ $= 0$ $\frac{62{\cdot}5}{80 - x} = 1{\cdot}5$ $x = 80 - \frac{62{\cdot}5}{1{\cdot}5}$ $x = 38{\cdot}3 = 38$ words $h\left(38\frac{1}{3}\right) = 1{\cdot}5\left(38\frac{1}{3}\right)$ $+ 62{\cdot}5\ln\left(1 - \frac{38\frac{1}{3}}{80}\right) = 16{\cdot}73$ = 17 days	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Any correct differentiation $h(x) = 1{\cdot}5x + 62{\cdot}5\ln\left(1 - \frac{x}{80}\right)$ $h'(x) = 0$ *High Partial Credit:* Differentiation correct but un-simplified Value for x and stops

Q8	Model Solution – 40 Marks	Marking Notes
(a)	$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ At $x = 0$: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(0)^2}$ $= \frac{1}{\sqrt{2\pi}}(1)$ $\therefore$ $(0, \frac{1}{\sqrt{2\pi}})$ is the y intercept	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $x = 0$ Value for x substituted into $f(x)$ *High Partial Credit:* $\frac{1}{\sqrt{2\pi}}$ *Full credit* – 1: (0, 0·3989)
(b)	$\text{Area} = \left[(2)\left(\frac{1}{\sqrt{2\pi e}}\right)\right] = 0{\cdot}4839$ $= 0{\cdot}484$ Units^2	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* length $= 2$ Width $=$ [y co-ordinate] *High Partial Credit:* $\left[(1)(\frac{1}{\sqrt{2\pi e}})\right]$ *Full credit* –1: Area = $-0{\cdot}484$ *Zero Credit*: Integrating original function
(c)	$C(1, \frac{1}{\sqrt{2\pi e}})$ due to symmetry $f'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(-x)$ $At\ x = 1$: $f'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(1)^2}(-1) < 0$ $\left[= -\frac{1}{\sqrt{2\pi e}} \quad (-0{\cdot}\ 24197) \ < 0 \ \right]$ $\Longrightarrow$ Decreasing	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $x = 1$ identified Some correct differentiation Indicates significance of $\frac{dy}{dx} < 0$ *High Partial Credit:* Derivative found

(d)		
	$f'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(-x)$ $f''(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(-1) + (-x)\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(-x)$ $= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(x^2 - 1)$ $f''(-1) = 0$ as $1^2 - 1 = 0$ $\Longrightarrow$ point of inflection at $x = -1$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $f'(x)$ transferred or found Mention of $f''(x)$ Identifies $x = -1$ *Mid Partial Credit:* $f''(x)$ identified and some correct differentiation *High Partial Credit:* $f''(x)$ found **Note:** if the product rule **and** chain rule are not applied in finding $f''(x)$ then the candidate can be awarded mid partial credit at most

<table>
<tr><th>Q9</th><th>Model Solution – 55 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td colspan="2">
<table>
<tr><th>Step</th><th>0</th><th>1</th><th>2</th><th>3</th></tr>
<tr><td>Triangles Remaining</td><td>1</td><td>3</td><td>9</td><td>27</td></tr>
<tr><td>Fraction of Original Triangle Remaining</td><td>1</td><td>$\frac{3}{4}$</td><td>$\frac{9}{16}$</td><td>$\frac{27}{64}$</td></tr>
</table>
</td></tr>
<tr><td></td><td></td><td>Scale 10C (0, 4, 8, 10)
Low Partial Credit:
One correct entry

High Partial Credit:
Three correct entries

Full credit –1:
Answers as decimals</td></tr>
<tr><td>(b)
(i)</td><td>3^n</td><td>Scale 5B (0, 2, 5)
Partial Credit:
$3n$ written
n^3 written

Full credit –1:
3^{n-1} written</td></tr>
<tr><td>(b)
(ii)</td><td>$3^k > 1{,}000{,}000{,}000$
$\log_3 3^k > \log_3 1\,000\,000\,000$
$k\log_3 3 \quad > \log_3 1\,000\,000\,000$
$k > \log_3 1 \times 10^9$
$k > 18{\cdot}863$
$k = 19$</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
$3^k > 1{,}000{,}000{,}000$

High Partial Credit:
Inequality with k not written as an index

Note: if $3k$ or k^3 from above used fully here then award low partial credit at most</td></tr>
</table>

(c) (i)	$\left(\frac{3}{4}\right)^h < \frac{1}{100}$ $\ln\left(\frac{3}{4}\right)^h < \ln\frac{1}{100}$ $h\ln\left(\frac{3}{4}\right) < \ln\frac{1}{100}$ $h > \frac{\ln\frac{1}{100}}{\ln\left(\frac{3}{4}\right)}$ $h > 16{\cdot}007$ $\Rightarrow h = 17$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Correct answer without work $\left(\frac{3}{4}\right)^h$ or candidates ratio to the power of h $r = \frac{3}{4}$ Lists two or more terms *High Partial Credit:* Inequality with h not written as an index *Full credit* –1: $\left(\frac{3}{4}\right)^{h-1} < \frac{1}{100}$ and finishes correctly
(c) (ii)	$\lim\limits_{n\to\infty}\left(\frac{3}{4}\right)^n = 0$ $\Longrightarrow \text{Fraction remaining} = 0$	**Scale 5B (0, 2, 5)** *Partial Credit:* $\lim\limits_{n\to\infty}$ Some use of $\frac{3}{4}$ *Full Credit*: Correct answer without work $\frac{1}{\infty}$ or equivalent
(d) (i)		

Step	0	1	2	3	4
Perimeter	3	$\frac{9}{2}$	$\frac{27}{4}$	$\frac{81}{8}$	$\frac{243}{16}$

Scale 10C (0, 4, 8, 10)
Low Partial Credit:
One correct entry
All numerators correct with all incorrect denominators
All denominators correct with all incorrect numerators

High Partial Credit:
Two correct entries

(d) (ii)	Pattern: $\frac{3^1}{2^0}, \frac{3^2}{2^1}, \frac{3^3}{2^2} \ldots\ldots\ldots \frac{3^{n+1}}{2^n}$ $\therefore$ step $35 = \frac{3^{36}}{2^{35}}$ $= 4\,368\,329$ **Or** $T_{35} = \left(\frac{9}{2}\right)\left(\frac{3}{2}\right)^{34} = 4\,368\,329$ **Or** $T_{35} = (3)\left(\frac{3}{2}\right)^{35} = 4\,368\,329$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Pattern identified Recognises $r = \frac{3}{2}$ Some relevant substitution into $T_n = ar^{n-1}$ $a = 3$ or $a = 4 \cdot 5$ *High Partial Credit:* Step $35 = \frac{3^{36}}{2^{35}}$ or equivalent *Full credit* −1: $T_{35} = (3)\left(\frac{3}{2}\right)^{34}$
(d) (iii)	Area = 0 $\lim_{n\to\infty}\left(\frac{3^{n+1}}{2^n}\right) = \infty$ $\Rightarrow$ Perimeter $\to \infty$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $\lim_{n\to\infty}\left(\frac{3^{n+1}}{2^n}\right)$ or equivalent Area is getting smaller Perimeter is increasing *High Partial Credit:* Area approaches 0 **Perimeter** $\to \infty$ identified Area is getting smaller and Perimeter is increasing

Blank Page

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2018

Marking Scheme

Mathematics

Higher Level

Paper 2

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	
5 mark scales		0, 2, 5	0, 2, 4, 5		
10 mark scales			0, 3,7, 10	0, 3, 5, 8, 10	
15 mark scales			0, 4, 11, 15	0, 4, 7, 11, 15	
20 mark scales			0, 7, 13, 20	0, 5, 10, 15, 20	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

Marking Scheme

Section A

Question 1

(a)	15C
(b)	10D

Question 2

(a)	5B
(b)(i)	10C
(b)(ii)	5C
(b)(iii)	5C

Question 3

(a)(i)	15C
(a)(ii)	5B
(b)	5C

Question 4

(a)	20C
(b)	5C

Question 5

(a)	10C
(b)	10D
(c)	5C

Question 6

(a)	15D
(b)	10C

Section B

Question 7

(a)	10D
(b)(i)	10C
(b)(ii)	10D
(c)	15C
(d)	5B

Question 8

(a)(i)	20D
(a)(ii)	15C
(b)(i)	5C
(b)(ii) (iii)	10D
(c)	10D

Question 9

(a)	10C
(b)(i)	5C
(b)(ii)	5C
(b)(iii)	10C
(b)(iv)	5B
(c)	5C

Note: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Throughout the scheme indicate by use of * where an arithmetic error occurs.

Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

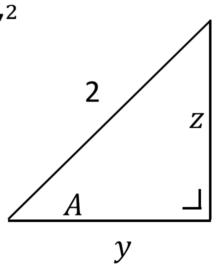
Q1	Model Solution – 25 Marks	Marking Notes
(a)	$\frac{1}{20}(9000)+\frac{1}{10}(7000)+\frac{1}{4}(3000)$ $= 1900$ $E(x)=2000-1900=100$ **Or** $E(x)=\frac{1}{20}(-7000)+\frac{1}{10}(-5000)+\frac{1}{4}(-1000)+\frac{3}{5}(2000)$ $=-350-500-250+1200=100$ So expected gain for organisers of competition and therefore a loss for Mary of 100	**Scale 15C (0, 4, 11, 15)** *Low Partial Credit:* $E(x)$ partially formulated (1 or 2 terms) *High Partial Credit:* $E(x)$ fully formulated (sum of all three/all four terms)

(b)	$\frac{1}{20}(9000+x)+\frac{1}{10}(7000+x)$ $+\frac{1}{4}(3000+x)=2000$ $\left(1900+\frac{8}{20}x\right)=2000$ $\frac{8}{20}x=100$ $x=250$ **Or** From **(a)** to break even it will take €100. $\frac{x}{20}+\frac{x}{10}+\frac{x}{4}=100$ $\frac{x+2x+5x}{20}=100$ $\frac{8}{20}x=100$ $x=250$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Any relevant use of x, excluding $(9000+x)$ *Mid Partial Credit:* $E(x)$ fully formulated (LHS). $\left(1900+\frac{8}{20}x\right)$ or equivalent and stops. $\frac{x}{20}+\frac{x}{10}+\frac{x}{4}$ *High Partial Credit* Relevant equation in x
	Or $E(x)=\frac{1}{20}(-7000-x)$ $+\frac{1}{10}(-5000-x)$ $+\frac{1}{4}(-1000-x)+\frac{3}{5}(2000)=0$ $-7000-x-10\,000-2x-5000-5x$ $+24\,000=0$ $2000=8x\Rightarrow 250=x$	*Low Partial Credit:* Any relevant use of x e.g. $(-7000+x)$ *Mid Partial Credit:* $E(x)$ fully formulated (LHS). $\left(100-\frac{8}{20}x\right)$ or equivalent and stops. *High Partial Credit* Relevant equation in x

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$P(z < z_1) = 0{\cdot}67$ $z = 0{\cdot}44$	**Scale 5B (0, 2, 5)** *Partial Credit:* $P(z < z_1) = 0{\cdot}67$
(b) (i)	Mary Maths $\frac{65-70}{15} = -\frac{1}{3}$ Mary English $\frac{68-72}{10} = -\frac{2}{5}$ $-\frac{1}{3} > -\frac{2}{5}$ Mary did better in Maths Justification: $-\frac{1}{3} > -\frac{2}{5}$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* Relevant formula with some correct substitution $\frac{65-70}{15}$ or $\frac{68-72}{10}$. *High Partial Credit:* $\frac{65-70}{15}$ and $\frac{68-72}{10}$
(b) (ii)	$P(z > z_1) = 0{\cdot}15$ $z = \frac{x-72}{10} = 1{\cdot}04$ $x = 82{\cdot}4\%$ $x = 83$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* $0{\cdot}15$ $1{\cdot}04$ Relevant formula with some correct substitution *High Partial Credit:* Relevant equation in x

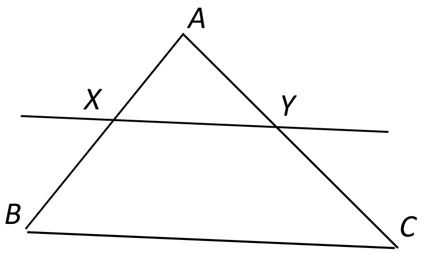
(b) (iii)	82 is 1 st. dev. above mean $\Rightarrow \approx \frac{68}{2}\%$ above 52 is 2 st. dev. below mean $\Rightarrow \approx \frac{95}{2}\%$ below $34 + 47{\cdot}5 = 81{\cdot}5\%$ **Or** From tables: 82 is 1 deviation off mean $\Rightarrow \frac{0{\cdot}6826}{2} = 0{\cdot}3413$ 52 is 2 dev. off mean $\Rightarrow \frac{0{\cdot}9544}{2} = 0{\cdot}4772$ $0{\cdot}3413 + 0{\cdot}4772 = 0{\cdot}8185 = 81{\cdot}85\%$ **Or** $z = \frac{52-72}{10} = -2$ $z = \frac{82-72}{10} = 1$ $P(-2 < z < 1)$ $P(z < 1) - [1 - P(z < 2)]$ $0{\cdot}8413 - [1 - 0{\cdot}9772]$ $= 0{\cdot}8185$ $= 81{\cdot}85\%$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* Evidence of relevant linking of deviation and mean. $\frac{68}{2}$ or $\frac{95}{2}$ $\frac{52-72}{10}$ or $\frac{82-72}{10}$ $\frac{0{\cdot}6826}{2}$ or $\frac{0{\cdot}9544}{2}$ *High Partial Credit:* $\frac{68}{2}$ and $\frac{95}{2}$ $\frac{52-72}{10}$ and $\frac{82-72}{10}$ $\frac{0{\cdot}6826}{2}$ and $\frac{0{\cdot}9544}{2}$

Q3	Model Solution – 25 Marks	Marking Notes
(a) **(i)**	$10^5 \times 1$ or $100\,000$	**Scale 15C (0, 4, 11, 15)** *Low Partial Credit:* Some use of 10. Identifies that 5 other digits are required to complete code. *High Partial Credit* 9^5 or equivalent 10^6
(a) **(ii)**	$1 \times 10 \times 10 + 10 \times 1 \times 10 + 10 \times 10 \times 1$ $3 \times 10 \times 10$ or 3×10^2 or 300	**Scale 5B (0, 2, 5)** *Partial Credit:* 10 × 10
(b)	$\dfrac{(n+3)!\ (n+2)!}{(n+1)!\,(n+1)!} =$ $(n+3)(n+2)(n+2) =$ $n^3 + 7n^2 + 16n + 12$ **Or** $\dfrac{(n+3)!\,(n+2)!}{(n+1)!\,(n+1)!} = an^3 + bn^2 + cn + d$ $n = 0 \rightarrow \dfrac{3!\,.2!}{1!\,1!} = 12 = d$ $n = 1 \rightarrow a + b + c + d = 36$ $n = 2 \rightarrow 8a + 4b + 2c + d = 80$ $n = 3 \rightarrow 27a + 9b + 3c + d = 150$ Solving the simultaneous equations $a = 1, b = 7, c = 16, d = 12$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* Factorial expansion (e.g. $(n+3)! = (n+3)(n+2)(n+1) \ldots \ldots \ldots .1)$ Effort at a numerical value for *n* on both LHS and RHS (method 2) *High Partial Credit:* $(n+3)(n+2)(n+2)$ Four simultaneous equations

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$2x = 150 + 360n$ or $2x = 210 + 360n$ $x = 75 + 180n$ $\quad x = 105 + 180n$ $n = 0 \Rightarrow x = 75°$ $\quad n = 0 \Rightarrow x = 105°$ $n = 1 \Rightarrow x = 255°$ $\quad n = 1 \Rightarrow x = 285°$	**Scale 20C (0, 7, 13, 20)** *Low Partial Credit:* 30° or 150° or 210° *High Partial Credit:* 2 relevant values of x
(b)	$2^2 = y^2 + z^2$ $z = \sqrt{4 - y^2}$ $\sin 2A = 2\sin A\cos A$ $2\left(\frac{\sqrt{4 - y^2}}{2}\right)\left(\frac{y}{2}\right)$ $= \frac{y\sqrt{4-y^2}}{2}$  **Or** $\sin 2A = \frac{2\tan A}{1 + \tan^2 A}$ $\frac{2\frac{\sqrt{4-y^2}}{y}}{1+\frac{4-y^2}{y^2}} = \frac{2y\sqrt{4-y^2}}{y^2+4-y^2} = \frac{y\sqrt{4-y^2}}{2}$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* $\sqrt{4 - y^2}$ $2\sin A\cos A$ without substitution $\sin 2A$ expressed in $\tan A$ format Relevant labelled diagram (2, y, A) *High Partial Credit:* Substitution for $\sin A$ or $\cos A$ in formula $\sin A = \left(\frac{\sqrt{4-y^2}}{2}\right)$ $\tan A = \frac{\sqrt{4-y^2}}{y}$

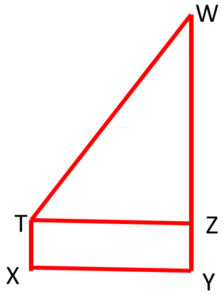
Q5	Model Solution – 25 Marks	Marking Notes
(a)	$2(-2) + 3(1) + 1 = 0$ or $-4 + 3 + 1 = 0$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* Substitution for x **or** y in equation of line *High Partial Credit:* Substitution for x **and** y in eq. of line (LHS when no indication of 0)
(b)	Slope of *m* or *n* = $\frac{-2}{3}$ Slope of *AB* is $\frac{3}{2}$ and (−2, 1) is on *AB* $y - 1 = \frac{3}{2}\left(x - (-2)\right)$ equation of *AB* is $3x - 2y + 8 = 0$ Solve for (x, y) between $3x - 2y + 8 = 0$ and $2x + 3y - 51 = 0$ $n \cap AB = (6, 13) = B$ **Or** coordinates of $B(x, y)$ $\lvert AB\rvert = \sqrt{(x+2)^2 + (y-1)^2}$ Perp. distance (−2, 1) to $2x + 3y - 51 = 0$ $\left\lvert\frac{-4+3-51}{\sqrt{13}}\right\rvert = \frac{52}{\sqrt{13}} = 4\sqrt{13}$ $\therefore (x+2)^2 + (y-1)^2 = (4\sqrt{13})^2$ Substituting $x = \frac{1}{2}(-3y + 51)$ $(\frac{-3y+55}{2})^2 + (y-1)^2 = (4\sqrt{13})^2$ $13y^2 - 338y + 2197 = 0$ $y^2 - 26y + 169 = 0$ $(y - 13)^2 = 0 \rightarrow y = 13$ $n \cap AB = (6, 13) = B$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Slope of *AB* Equation of line formula with some substitution *Mid Partial Credit:* Equation of *AB* *High Partial Credit:* Effort at finding intersection of lines **Note**: Point of intersection, found correctly, of *n* and a relevant *AB* (with errors) merits Mid Partial Credit at least. Method 2 *Low Partial Credit:* Perpendicular distance formula with some substitution Distance formula with some substitution *Mid Partial Credit:* Quadratic equation in x and y *High Partial Credit:* Quadratic equation in either x or y

(c)	$\overrightarrow{AB} = x$ up 8 and y up 12 Centre of *s* is $\frac{1}{8}(8) - 2 = -1 = h$ and $\frac{1}{8}(12) + 1 = 2{\cdot}5 = k$ Eqn *s*: $(x+1)^2 + (y-2{\cdot}5)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* 8 up or 12 up Indication $4\sqrt{13}$ from(b) of relevance *High Partial Credit:* Centre and radius of circle
	Or	
	$s \cap t$ $\left(\frac{3(-2)+1(6)}{3+1}, \frac{3(1)+1(13)}{3+1}\right) = (0, 4)$ Centre *s*: $\left(\frac{0-2}{2}, \frac{4+1}{2}\right) = (-1,\ 2{\cdot}5)$ Radius : distance $(-1, 2{\cdot}5)$ to either $(-2, 1)$or $(0, 4)$ or calculated otherwise $\sqrt{3{\cdot}25}$ or $\frac{\sqrt{13}}{2}$ $(x+1)^2 + (y-2{\cdot}5)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$	*Low Partial Credit:* Some relevant use of 1 : 3 Midpoint of AB found once but no further work of relevance Formula with some relevant substitution *High Partial Credit:* Centre and radius of circle
	Or	
	using ratio 1 : 7 centre *s*: $\left(\frac{1(6)+7(-2)}{1+7}, \frac{1(13)+7(1)}{1+7}\right) = (-1, 2{\cdot}5)$ Radius as above or $\mathbf{\frac{1}{8}}|AB|= \mathbf{\frac{\sqrt{13}}{2}}$ $(x+1)^2 + (y-2{\cdot}5)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$	*Low Partial Credit:* Some relevant use of 1 : 7 Formula with some relevant substitution *High Partial Credit:* Centre and radius of circle

Q6	Model Solution – 25 Marks	Marking Notes
(a)	*Diagram*:  Given: A triangle *ABC* and a line *XY* parallel to *BC* which cuts *AB* in the ratio $s : t$ where $s, t \in \mathbb{N}$. *To Prove*: $[AY] : [YC] = s : t$ *Construction*: Divide [*AB*] into $s + t$ equal parts, *s* of them lying along [*AX*] and *t* of them lying along [*XB*]. Through each point of division draw a line parallel to [*BC*] *Proof*: By a previous theorem the parallel lines cut off segments of equal length along [*AC*]. Therefore [*AC*] is divided into $s + t$ equal parts with *s* of them forming [*AY*] and *t* of them forming [*YC*]. Let *k* be the length of one segment on [*AC*]. $[AY] : [YC] = ks : kt \quad = s : t$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Relevant diagram drawn *Mid Partial Credit:* Construction clearly indicated *High Partial Credit:* Proof missing 1 relevant step

(b)		
	$|XY| = \sqrt{4^2 + 3^2} = 5$ $|ZC| = 5$ $|BZ| = 10\text{cm}$ **Or** $\frac{8}{4}$ or $\frac{2}{1} = \frac{|BZ|}{5} \rightarrow |BZ| = 10\text{cm}$ **Or** $\frac{4}{12} = \frac{5}{5 + |BZ|}$ $4|BZ| + 20 = 60 \rightarrow |BZ| = 10\text{ cm}$ Similarly: $\frac{3}{9} = \frac{5}{5+|BZ|}$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* $|XY|$ or $|BX|$ or $|CY|$ found Pythagoras with some substitution *High Partial Credit:* $|ZC|$ or $|BC|$ found Ratios formulated with $|BZ|$ the sole unknown

Q7	Model Solution – 50 Marks	Marking Notes
(a)	$V = \frac{4}{3}\pi 3^3 = 36\pi = 113{\cdot}1$ $\frac{113{\cdot}1(1-1{\cdot}75^5)}{1-1{\cdot}75} = 2324{\cdot}29$ $= 2324$ **or** Volume A = $113{\cdot}1$ Volume B = $197{\cdot}925$ Volume C = $346{\cdot}36875$ Volume D = $606{\cdot}1453125$ Volume E = $1060{\cdot}754296875$ Total: 2324·293359375= 2324	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Volume formula with some substitution *Mid Partial Credit:* Volume of 2 spheres GP formula with some substitution *High Partial Credit:* Volume of 5 spheres G P formula fully substituted
(b) **(i)**	$4\pi r^2 = 503 \Rightarrow r = \sqrt{\frac{503}{4\pi}} = 6{\cdot}33$ Height =$120 - 2(6{\cdot}33) = 107{\cdot}3$ **Or** $\frac{4}{3}\pi r^3 = 1060{\cdot}754 \text{ from(a)}$ $r = 6{\cdot}326$ Height : $120 - 2(6{\cdot}326) = 107{\cdot}348$=$107{\cdot}3$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* $4\pi r^2 = 503$ $\frac{4}{3}\pi r^3$ = volume from (a) *High Partial Credit:* *r* found
(b) **(ii)**	A: $\pi 1^2 h = 71{\cdot}3\pi \Rightarrow h = 71{\cdot}3$ Height difference: $107{\cdot}3 - 71{\cdot}3 = 36$ $\frac{36}{4} = 9$ step up in each bar. **Or** T_5 = $71{\cdot}3 + 4d = 107{\cdot}3 \rightarrow d = 9$ Height of each bar (in cm) $71{\cdot}3,\ 80{\cdot}3,\ 89{\cdot}3,\ 98{\cdot}3,\ 107{\cdot}3$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Vol formula with some substitution $\pi r^2 h = 71{\cdot}3\pi$ *Mid Partial Credit:* Height of bar A *High Partial Credit:* Difference in height between bar A and bar E

(c)	$150 - (20 + 20 + 9(2)) = 92$ $\frac{92}{8}$ cm or 11·5 cm	**Scale 15C (0, 4, 11, 15)** *Low Partial Credit:* Recognises 8 equal divisions Indicates subtraction of one relevant length 9×2 *High Partial Credit:* $150 - 40 - 18$ or equivalent
(d)	$V_B = 1{\cdot}75\left(\frac{4}{3}\pi 3^3\right) = 63\pi$ $V_B = \frac{4}{3}\pi r^3 = 63\pi \Rightarrow r_b = 3{\cdot}62$ cm  $\lvert XY\rvert = 1 + 11{\cdot}5 + 1 = 13{\cdot}5$ $\lvert ZW\rvert = (9 - 3) + 3{\cdot}62 = 9{\cdot}62$ $\lvert TW\rvert = \sqrt{13{\cdot}5^2 + 9{\cdot}62^2} = 16{\cdot}576$ **Or** $\tan\angle WTZ = \frac{9{\cdot}62}{13{\cdot}5} \to \lvert\angle WTZ\rvert = 35{\cdot}459°$ $\cos\angle WTZ = \frac{13{\cdot}5}{\lvert TW\rvert} \to \lvert TW\rvert = 16{\cdot}576$ The rod is: $\lvert TW\rvert - 3 - 3{\cdot}62$ $= 16{\cdot}576 - 3 - 3{\cdot}62 = 9{\cdot}95$ $\lvert TW\rvert = 10$	**Scale 5B (0, 2, 5)** *Partial Credit:* V_B formulated with some substitution $\lvert XY\rvert$ formulated $\lvert TW\rvert$ evaluated rod $= \lvert TW\rvert - r_b - r_a$ formulated with 2 relevant values

Q8	Model Solution – 60 Marks	Marking Notes
(a) **(i)**	$z_1 = \frac{4{\cdot}6 - 4{\cdot}64}{\frac{0{\cdot}12}{\sqrt{10}}} = -1{\cdot}05409$ $z_2 = \frac{4{\cdot}7 - 4{\cdot}64}{\frac{0{\cdot}12}{\sqrt{10}}} = 1{\cdot}581138$ $p(-1{\cdot}05 < z < 1{\cdot}58)$ $= 0{\cdot}9429 - (1 - 0{\cdot}8531)$ $= 0{\cdot}796$ or 79·6%	**Scale 20D (0, 5, 10, 15, 20)** *Low Partial Credit:* z_1 formulated with some correct substitution z_2 formulated with some correct substitution *Mid Partial Credit:* z_1 and z_2 fully substituted *High Partial Credit:* –1·05 and 1·58 or equivalents
(a) **(ii)**	Confidence Interval: $\hat{p} \pm 1{\cdot}96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $0{\cdot}81 - 1{\cdot}96\sqrt{\frac{0{\cdot}81 \times 0{\cdot}19}{400}} \leq p$ $\leq 0{\cdot}81 + 1{\cdot}96\sqrt{\frac{0{\cdot}81 \times 0{\cdot}19}{400}}$ $0{\cdot}77155 \leq p \leq 0{\cdot}848445$ $0{\cdot}77 \leq p \leq 0{\cdot}85$	**Scale 15 C (0, 4, 11, 15)** *Low Partial Credit:* CI formulated with some correct substitution $1{\cdot}96$ $\hat{p} \pm \frac{1}{\sqrt{n}}$ incomplete or completed *High Partial Credit:* CI fully substituted

(b) (i)

Statement	Always True	Sometimes True	Never True
1. When forming confidence intervals (for fixed *n* and $\hat{p}$), an increased confidence level implies a wider interval.	✓		
2. As the value of $\hat{p}$ increases (for fixed *n*), the estimated standard error of the population proportion increases.		✓	
3. As the value of $\hat{p}(1-\hat{p})$ increases (for fixed *n*), the estimated standard error of the population proportion increases.	✓		
4. As *n*, the number of people sampled, increases (for fixed $\hat{p}$), the estimated standard error of the population proportion increases.			✓

Scale 5C (0, 2, 4, 5)
Low Partial Credit:
Any 1 correct

High Partial Credit:
Any 2 correct

<table>
<tr>
<td>(b)
(ii)+
(iii)</td>
<td>$M = \hat{p} - \hat{p}^2$
$\frac{dM}{d\hat{p}} = 1 - 2\hat{p}$
$\frac{dM}{d\hat{p}} = 1 - 2\hat{p} = 0$
$\hat{p} = \frac{1}{2}$
$M_{max} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

Or
$f(p) = \hat{p} - \hat{p}^2 = -(\hat{p}^2\ \hat{p})$
$= -[(\hat{p}^2 - \hat{p} + (-\frac{1}{2})^2) - (-\frac{1}{2})^2]$
$= \frac{1}{4} - \left(\hat{p} - \frac{1}{2}\right)^2$
$\hat{p} = \frac{1}{2} \quad M_{max} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$1{\cdot}96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
$= 1{\cdot}96\sqrt{\frac{1}{4n}} = 0{\cdot}03464 = 3{\cdot}46\%$</td>
<td>Scale 10D (0, 3, 5, 8, 10)
Low Partial Credit:
Any relevant calculus
$\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{800}}$
Effort at completing the square

Mid Partial Credit
$\hat{p} = \frac{1}{2}$ or equivalent

High Partial Credit:
Maximum value</td>
</tr>
<tr>
<td>(c)</td>
<td>$20\,000 + \frac{20\,000(1{\cdot}01)}{1{\cdot}024} + \frac{20\,000(1{\cdot}01^2)}{1{\cdot}024^2} + \frac{20\,000(1{\cdot}01^3)}{1{\cdot}024^3} + \cdots + \frac{20\,000(1{\cdot}01^{25})}{1{\cdot}024^{25}}$

$20\,000\left[1 + \frac{1{\cdot}01}{1{\cdot}024} + \frac{1{\cdot}01^2}{1{\cdot}024^2} + \frac{1{\cdot}01^3}{1{\cdot}024^3} + \cdots + \frac{1{\cdot}01^{25}}{1{\cdot}024^{25}}\right]$
$a = 1,\ r = \frac{1{\cdot}01}{1{\cdot}024} = \frac{505}{512}\ , n = 26$

$20000\left[\frac{\left(1 - \frac{505^{26}}{512^{26}}\right)}{1 - \frac{505}{512}}\right] = €440\,132{\cdot}40$</td>
<td>Scale 10D (0, 3, 5, 8, 10)
Low Partial Credit:
20000(1·01) or $\frac{20000}{1{\cdot}024}$

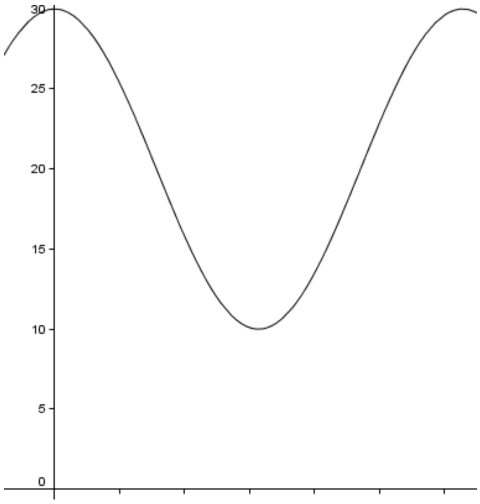
Mid Partial Credit:
$\frac{20\,000(1{\cdot}01)}{1{\cdot}024}$ or similar term
Correctly handles inflation element or completes correctly present values element and finishes

High Partial Credit:
GP with a, r and n identified

Note: Treat $n = 25$ as a misreading</td>
</tr>
</table>

Q9	Model Solution – 40 Marks	Marking Notes
(a)	$$\frac{10}{\sin 15} = \frac{30}{\sin x}$$ $$\sin x = \frac{30\ \sin 15}{10}$$ $$\sin x = 0{\cdot}77645$$ $$x = 51°$$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* Sine rule formulated with some substitution *High Partial Credit:* $\sin x$
(b) (i)	period = 2π Range = [10, 30]	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* Period **or** range correct *High Partial Credit:* Period correct and range partly correct Period **and** range in incorrect order
(b) (ii)	see table below	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* 1 correct new value *High Partial Credit:* 2 correct new values

α	$0°$	$90°$	$180°$	$270°$	$360°$
$f(\alpha)$ (cm)	30	18·28	10	18·28	30

<table>
<tr><th>Q9</th><th colspan="2">Marking Notes</th></tr>
<tr><td>(b)
(iii)</td><td>
</td><td>Scale 10C (0, 3, 7, 10)
Low Partial Credit:
1 point from table plotted

High Partial Credit:
3 points from table plotted</td></tr>
<tr><td>(b)
(iv)</td><td>Answer: diagram 2

refer to the steepness of their graph at the three corresponding points
or
rely on the original geometry of the situation: the closer ∠CDO is to a right angle the more the connecting rod will get pulled or pushed by a small change in the crank angle</td><td>Scale 5B (0, 2, 5)
Partial Credit:
Diagram 2 identified but without reason or with invalid reason</td></tr>
</table>

(c)	$r^2 = 36^2 + (31+r)^2$ $- 2(36)(31+r)\cos 10°$ $r^2 = 1296 + 961 + 62r + r^2$ $-(2232\cos 10° - 72r\cos 10°)$ $8{\cdot}906r = 58{\cdot}91$ $r = 6{\cdot}62$ $r = 7$ **Or** $\lvert BX\rvert^2 = 36^2 + 31^2 - 2 \times 36 \times 31\cos 10°$ $\lvert BX\rvert^2 = 58{\cdot}91$ $\lvert BX\rvert = 7{\cdot}675$ $\dfrac{\sin 10°}{7{\cdot}675} = \dfrac{\sin \angle BXA}{36}$ $\angle BXA = 125{\cdot}462° \Rightarrow \angle BXO = 54{\cdot}53795°$ ΔBXO is isosceles $\Rightarrow \angle BOX = 70{\cdot}924°$ $\dfrac{\sin 70{\cdot}924°}{7{\cdot}675} = \dfrac{\sin 54{\cdot}53795°}{r}$ $r = 6{\cdot}6145$ $r = 7$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* Cosine rule formulated with some substitution $(31+r)$ *High Partial Credit:* Relevant equation in r