Transition Year Hons Maths - Problem Set 1

1. Simplify the following

(i) $\dfrac{x^2+2xy+y^2}{x^2-y^2}\times\dfrac{3x-3y}{4x+4y}$

$= \dfrac{(x+y)(x+y)}{(x+y)(x-y)} \times \dfrac{3(x-y)}{4(x+y)}$

$= \dfrac{3}{4}$

(ii) $\sqrt{\dfrac{x^3+6x^2+9x}{x}}$

$= \sqrt{x^2+6x+9}$

$= \sqrt{(x+3)(x+3)}$

$= \sqrt{(x+3)^2} = x+3$

(iii) $\dfrac{x^{-\frac{1}{2}}+x^{\frac{1}{2}}}{x^{-\frac{1}{2}}-x^{\frac{3}{2}}}$

$= \dfrac{x^{\frac{1}{2}}\left(x^{-\frac{1}{2}}+x^{\frac{1}{2}}\right)}{x^{\frac{1}{2}}\left(x^{-\frac{1}{2}}-x^{\frac{3}{2}}\right)}$

$= \dfrac{x^0+x}{x^0-x^2} = \dfrac{1+x}{1-x^2}$

$= \dfrac{1+x}{(1+x)(1-x)} = \dfrac{1}{1-x}$

2. Simplify

(i) $(a-b)^2-(a+b)^2$

$= a^2\ 2ab+b^2-(a^2+2ab+b^2)$

$= a^2-2ab+b^2-a^2-2ab-b^2$

$= -4ab$

(ii) Hence simplify $\left(\sqrt{x}-\sqrt{y}\right)^2-\left(\sqrt{x}+\sqrt{y}\right)^2$

Let $\sqrt{x}=a$ and $\sqrt{y}=b$

From (i)

$\left(\sqrt{x}-\sqrt{y}\right)^2-\left(\sqrt{x}+\sqrt{y}\right)^2 = -4\sqrt{x}\sqrt{y}$

3. Factorise fully each of the following:

(i) $5x^2+18x-8$

$= (5x-2)(x+4)$

(ii) $x^2y-y^3+x^3-xy^2$

$= y(x^2-y^2)+x(x^2-y^2)$

$= (x^2-y^2)(x+y)$

$= (x-y)(x+y)(x+y)$

(iii) $a^2-2ab+b^2-4c^2$

$\underbrace{a^2-2ab+b^2}-4c^2$

$(a-b)^2-4c^2$

$= (a-b-2c)(a-b+2c)$

(iv) $x^2+3px-4p^2$

$x^2+3px-4p^2$

$(x+4p)(x-p)$

4. Use factorisation to solve the following equations:

(i) $x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x = -2 \quad | \quad x = -1$

(ii) $6x^2 - 5x - 4 = 0$

$(3x-4)(2x+1) = 0$

$3x - 4 = 0 \quad | \quad 2x + 1 = 0$

$x = \frac{4}{3} \quad | \quad x = -\frac{1}{2}$

(iii) $x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0 \quad | \quad x = 4$

5. Use the quadratic formula to solve the following equations to 2 decimal places.

(i) $x^2 + 7x + 9 = 0$

$a = 1,\ b = 7,\ c = 9$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(9)}}{2(1)} = \frac{-7 \pm \sqrt{13}}{2}$$

$$x = \frac{-7 \pm 3.61}{2} \Rightarrow x = 5.31$$

$x = 1.70$

(ii) $4x^2 - x - 7 = 0$

$a = 4,\ b = -1,\ c = -7$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-7)}}{2(4)} = \frac{-1 \pm \sqrt{113}}{8}$$

$$x = \frac{-1 \pm 10.63}{8} \Rightarrow x = -1.45$$

$x = 1.20$

6. Show that $(a+b)^2 - 2ab = a^2 + b^2$

$(a+b)^2 - 2ab$

$= a^2 + \cancel{2ab} + b^2 - \cancel{2ab}$

$= a^2 + b^2$

8. Show, using multiplication, that

$(x+y)(x^2 - xy + y^2) = x^3 + y^3$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - \cancel{x^2y} + \cancel{xy^2} + \cancel{x^2y} - \cancel{xy^2} + y^3$

$= x^3 + y^3$

7. *Solve the following simultaneous equations:

$$\left.\begin{aligned} x + 2y + 4z &= 7 \\ x + 3y + 2z &= 1 \\ -y + 3z &= 8 \end{aligned}\right\}$$

$$\left.\begin{aligned} x + 2y + 4z &= 7 \\ -x - 3y - 2z &= -1 \end{aligned}\right\} ① - ②$$

$-y + 2z = 6 \quad \text{....} ④$

$$\left.\begin{aligned} -y + 3z &= 8 \\ y - 2z &= -6 \end{aligned}\right\} ③ - ④$$

$\boxed{z = 2}$

$-y + 3z = 8$

$-y + 3(2) = 8$

$-y = 2$

$\boxed{y = -2}$

$x + 2y + 4z = 7$

$x + 2(-2) + 4(2) = 7$

$x + 4 = 7$

$\boxed{x = 3}$

Answers: $x = 3$, $y = -2$, $z = 2$