TY Honours Maths - Worksheet No.10

1. Given that $\sqrt{4x^2+12x+9} = ax+b$, find the values of the constants a and b.

square both sides

$$\left(\sqrt{4x^2+12x+9}\right)^2 = (ax+b)^2$$

$$4x^2+12x+9 = a^2x^2+2abx+b^2$$

$a^2=4$ | $2ab=12$

$a=2$ | $b=3$

$ax+b = 2x+3$

2. Express $\frac{3}{\sqrt{20}}+\frac{8}{\sqrt{45}}$ in the form $\frac{k\sqrt{m}}{n}$ where k, m and $n \in \mathbb{N}$

$$\frac{3}{\sqrt{4}\sqrt{5}} + \frac{8}{\sqrt{9}\sqrt{5}} = \frac{3}{2\sqrt{5}} + \frac{8}{3\sqrt{5}}$$

$$= \frac{9\sqrt{5}+16\sqrt{5}}{30}$$

$$= \frac{25\sqrt{5}}{30} = \frac{5\sqrt{5}}{6}$$

3. Explain briefly what is meant by the factor theorem.

If $x-k$ is a factor of the polynomial $f(x)$, then $x=k$ is a root and $f(k)=0$

4. Find the value of k if the equation $k^2x^2+2(k+1)x+4=0$ has equal roots.

If equal $\Rightarrow b^2-4ac=0$

$a=k^2,\ b=2k+2,\ c=4$

$b^2-4ac = (2k+2)^2-4(k^2)(4)=0$

$\Rightarrow 4k^2+8k+4-16k^2=0$

$12k^2-8k-4=0$

$12k^2-8k-4=0$

$4(3k^2-2k-1)=0$

$(3k+1)(k-1)=0$

$k=-\frac{1}{3}$ | $k=1$

5. Given that $x-1$ is a factor of $2x^3+tx^2+4x+2t$, find the value of t.

If $x-1$ is a factor

$\Rightarrow x=1$ is a root $\Rightarrow f(1)=0$

$f(1) = 2(1)^3+t(1)^2+4(1)+2t=0$

$2+t+4+2t=0$

$3t=-6$

$t=-2$

6. Factorise fully

(i) x^4-x

$= x(x^3-1)$

$= x(x-1)(x^2+x+1)$

(ii) $3x^2+26x-9$

$= (3x-1)(x+9)$

7. Given that the quadratic equation $x^2 + 2tx - 2x + 2t + 1 = 0$ has equal roots,

(i) find the value of t where $t > 0$.

(ii) use this value of t to evalute the roots.

For equal roots $\Rightarrow b^2 - 4ac = 0$

$(2t-2)^2 - 4(1)(2t+1) = 0$

$4t^2 - 8t + 4 - 8t - 4 = 0$

$4t^2 - 16t = 0$

$4t(t-4) = 0$

$t = 0$ | $t = 4$

$t > 0 \Rightarrow t = 4$

$x^2 + 2(4)x - 2x + 2(4) + 1 = 0$

$x^2 + 6x + 9 = 0$

$(x+3)(x+3) = 0$

$x = -3$ | $x = -3$

8. Write down a quadratic equation that has roots of 2 and -3 in the form of $ax^2 + bx + c = 0$ where $a, b, c \in Z$.

$(x-2)(x+3)$

$= x^2 - 2x + 3x - 6$

$\Rightarrow \boxed{x^2 + x - 6 = 0}$

OR

$x^2 - (\text{Sum of r's})x + (\text{Prod of rs}) = 0$

9. Solve the following equation: $x^2 + 6x - 2 = 0$ leaving your answers in surd form:

$a = 1, b = 6, c = -2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2}$

$x = \dfrac{-6 \pm \sqrt{44}}{2} = -3 \pm \sqrt{11}$

10. Fill in the following table

$b^2 - 4ac$

Quadratic	Discriminant	Nature of roots
$x^2 + 6x + 9$	$6^2 - 4(1)(9)$ $36 - 36 = 0$	Roots are equal
$2x^2 + 3x + 2$	$3^2 - 4(2)(2)$ $9 - 16 = -7$	Roots are not real (imaginary)
$3x^2 + 6x + 2$	$6^2 - 4(3)(2)$ $36 - 24 = 12$	Roots are real + distinct