TY Hons Maths - Problem Set 2

1. Simplify the following

(i) $\dfrac{x^2-xy}{x^2-y^2}$ (ii) $\dfrac{x^2-2xy+y^2}{x^2-y^2}\times\dfrac{3x+3y}{4x-4y}$ (iii) $\dfrac{1}{x^2-4}-\dfrac{x}{x+2}$

(i) $\dfrac{x^2-xy}{x^2-y^2} = \dfrac{x(x-y)}{(x-y)(x+y)} = \dfrac{x}{x+y}$

(ii) $\dfrac{x^2-2xy+y^2}{x^2-y^2} \times \dfrac{3x+3y}{4x-4y}$

$= \dfrac{(x-y)(x-y)}{(x-y)(x+y)} \times \dfrac{3(x+y)}{4(x-y)}$

$= \dfrac{3}{4}$ which is a constant

(iii) $\dfrac{1}{x^2-4} - \dfrac{x}{x+2}$

$= \dfrac{1(x^2-4)-x(x-2)}{x^2-4}$

$= \dfrac{x^2-4-x^2+2x}{x^2-4}$

$= \dfrac{2x-4}{x^2-4}$

$= \dfrac{2(x-2)}{(x-2)(x+2)}$

$= \dfrac{2}{x+2}$

2. Factorise fully each of the following

(i) $3x^2-11x+6$ (ii) $x^2y-y^3+x^3-xy^2$ (iii) $8x^3-64y^3$

(i) $3x^2-11x+6$

$= (3x-2)(x-3)$

6	
1	6
2	3

(ii) $x^2y-y^3+x^3-xy^2$

$= y(x^2-y^2)+x(x^2-y^2)$

$= (x+y)(x^2-y^2)$

$= (x+y)(x+y)(x-y)$

(iii) $8x^3-64y^3$

$(2x)^3-(4y)^3$

$= (2x-4y)(4x^2+8xy+16y^2)$

3. Given that $x = \sqrt{a+1}$ and $y = \sqrt{a-1}$, evaluate $x^2 - y^2$.

$x = \sqrt{a+1} \qquad y = \sqrt{a-1}$

$$x^2 - y^2 = \left(\sqrt{a+1}\right)^2 - \left(\sqrt{a-1}\right)^2 = a+1-(a-1)$$

$$= a+1-a+1$$

$$= 2$$

---

4. Use factorisation to solve the following equations:

(i) $x^2 + x - 12 = 0$ (ii) $8x^2 - 10x - 3 = 0$ (iii) $x^2 - 3x = 0$

(i) $x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x - 4 = 0$ | $x + 3 = 0$

$x = 4$ | $x = -3$

(ii) $8x^2 - 10x - 3 = 0$

$(4x+1)(2x-3) = 0$

$4x + 1 = 0$ | $2x - 3 = 0$

$4x = -1$ | $2x = 3$

$x = -\frac{1}{4}$ | $x = \frac{3}{2}$

(iii) $x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$ | $x - 3 = 0$

$x = 3$

---

5. Find the discriminant of the following equations and hence state the *nature* of their roots.
(*Nature* means whether they are real, not real etc)

(i) $x^2 + 8x + 16 = 0$ (ii) $x^2 + 3x + 7 = 0$ (iii) $x^2 + 5x + 2 = 0$

Quadratic	$x^2+8x+16=0$	$x^2+3x+7=0$	$x^2+5x+2=0$
Discriminant $[b^2-4ac]$	$a=1, b=8, c=16$ $b^2-4ac = 8^2-4(1)(16)$ $= 64-64$ $= 0$	$a=1, b=3, c=7$ $b^2-4ac = 3^2-4(1)(7)$ $= 9-28$ $= -19$	$a=1, b=5, c=2$ $b^2-4ac = 5^2-4(1)(2)$ $= 25-8$ $= 17$
	Since $b^2-4ac = 0$ Roots are real + equal.	Since $b^2-4ac < 0$ $\Rightarrow$ roots are not real	Since $b^2-4ac > 0$ $\Rightarrow$ roots are real + different

7. Given that the quadratic equation $x^2 + 2tx - 2x + 2t + 1 = 0$ has equal roots,

(i) find the value of t where $t > 0$.

(ii) use this value of t to evalute the roots.

For equal roots, discriminant $= 0$

(i) $x^2 + 2tx - 2x + 2t + 1 = 0$

$a = 1$, $b = 2t - 2$, $c = 2t + 1$

$b^2 - 4ac = 0$

$(2t-2)^2 - 4(1)(2t+1) = 0$

$4t^2 - 8t + 4 - 8t - 4 = 0$

$4t^2 - 16t = 0$

$4t(t-4) = 0$

$t = 0 \mid t = 4$

(ii) For $t = 4$

$x^2 + 2tx - 2x + 2t + 1 = 0$

becomes

$x^2 + 6x + 9 = 0$

$(x+3)(x+3) = 0$

$x = -3 \mid x = -3$

roots: -3 and -3

8/

$4x^2 + 8x + 12 = a(x+b)^2 + c$

$= a(x^2 + 2bx + b^2) + c$

$= ax^2 + 2abx + ab^2 + c$

$a = 4$ | $2ab = 8$, $2(4)b = 8$, $b = 1$

$ab^2 + c = 12$

$4(1)^2 + c = 12$

$c = 8$

$\boxed{a = 4,\ b = 1,\ c = 8}$