TY Hons Maths - Problem Set No.7

Name of Student: ____________________ **For** ________________

1. Evaluate $\frac{2+\sqrt{5}}{2-\sqrt{5}}$

$= \frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}$

$= \frac{+2\sqrt{5}+2\sqrt{5}+5}{2^2-(\sqrt{5})^2}$

$= \frac{9+4\sqrt{5}}{-1} = -9-4\sqrt{5}$

2. Prove that $-k$ is a root of the function $f(x) = x^3 + (1-k^2)x + k$, where k is a constant.

$f(-k) = (-k)^3 + (1-k^2)(-k) + k$

$= -k^3 - k + k^3 + k$

$= 0$ (all cancel)

Since $f(-k) = 0 \Rightarrow -k$ is a root.

3. Find the real numbers a and b such that

$x^2 - 2x - 6 = (x+a)^2 + b$

$x^2 - 2x - 6 = x^2 + 2ax + a^2 + 6$

$2a = -2$ | $a^2 + 6 = -6$

$a = -1$ | $(-1)^2 + b = -6$

$b = -6 - 1$

$b = -7$

$\boxed{a=-1}$ $\boxed{b=-7}$

4. Simplify $\frac{x^3+8}{x+2} \div \frac{x^2-2x+4}{2x+4}$

$= \frac{x^3+8}{x+2} \times \frac{2x+4}{x^2-2x+4}$

$= \frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{x+2}} \times \frac{2\cancel{(x+2)}}{x^2-2x+4}$

$= 2$

5. Given that the quadratic equation $x^2 + 2tx - 2x + 2t + 1 = 0$ has equal roots,

(i) find the value of t where $t > 0$.

(ii) use this value of t to evalute the roots.

$x^2 + (2t-2)x + 2t + 1 = 0$

$a = 1,\ b = 2t-2,\ c = 2t+1$

Equal roots $\Rightarrow b^2 - 4ac = 0$

$(2t-2)^2 - 4(\ )(2t+1) = 0$

$4t^2 - 8t + 4 - 8t - 4 = 0$

$4t^2 - 16t = 0$

$4t(t-4) = 0$

$4t = 0$ | $t = 4$

$t = 0$

(ii) $t > 0 \Rightarrow t = 4$

$x^2 + (2t-2)x + 2t + 1 = 0$

$x^2 + 6x + 9 = 0$

$(x+3)(x+3) = 0$

$x = -3$ | $x = -3$

6. Solve the following systems of simultaneous equations: $\begin{cases} 2x^2 - y^2 = 14 \\ x - y = 1 \end{cases}$

Line: $x - y = 1$

$y = x - 1$

Curve: $2x^2 - y^2 = 14$

$2x^2 - (x-1)^2 = 14$

$2x^2 - (x^2 - 2x + 1) = 14$

$2x^2 - x^2 + 2x - 1 = 14$

$x^2 + 2x - 15 = 0$

$\Rightarrow x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x = -5$	$x = 3$
$y = x - 1$	$y = x - 1$
$y = -5 - 1$	$y = 3 - 1$
$y = -6$	$y = 2$
$(-5, -6)$	$(3, 2)$

7. Write down the cubic equation which has roots of -1, -2 and 3 and which contains the point $(0, -12)$.

Roots: -1 -2 and 3

Factors: $x+1$, $x+2$, $x-3$

$f(x) = k\left[(x+1)(x+2)(x-3)\right]$

$= k\left[(x+1)(x^2 - x - 6)\right]$

$= k\left[x^3 - 7x - 6\right]$

$(0, -12) \Rightarrow f(0) = -12$

$f(0) = k\left[0^3 - 7(0) - 6\right] = -12$

$-6k = -12$

$k = 2$

$f(x) = 2\left[x^3 - 7x - 6\right]$

$f(x) = 2x^3 - 14x - 12$

8. The function $f(x)=2x^2+8x-4$ can be expressed as $a(x+b)^2+c$, where $a,b,c \in Z$

(i) Find the values of a,b and c.

(ii) Hence, find the co-ordinates of the local minimum of the curve.

(iii) Solve the equation $f(x)=0$, writing your answers in surd form.

(iv) Where does the graph cut the y-axis?

(v) Draw a rough sketch of $f(x)$ on the graph paper given.

(i) $a(x+b)^2+c = 2x^2+8x-4$

$a(x^2+2bx+b^2)+c = 2x^2+8x-4$

$ax^2+2abx+ab^2+c = 2x^2+8x-4$

$a=2$

$2ab = 8$

$ab = 4$

$(2)b = 4$

$b = 2$

$a^2b+c = -4$

$(2)^2(2)+c = -4$

$8+c = -4$

$c = -12$

(ii) $f(x) = a(x+b)^2+c = 2(x+2)^2-12$

Minimum when $x = -2$

$f(-2) = 2(-2+2)^2-12 = -12$

minimum $= (-2,-12)$

(iii) $2(x+2)^2-12 = 0$

$(x+2)^2 = 6$

$x = -2 \pm \sqrt{6}$

(iv) At y-axis, $x = 0$

$f(0) = 2(0)^2+8(0)-4 = -4$ $(0,-4)$

(v)