TY Hons Maths – Homework No.8

Name of Student: ______________________ **For** ____________________

1. Factorise the following:

(i) $6xy + 3x^2y - 9x^2y^3$ $= 3xy(2 + x - 3xy^2)$	(iii) $6x^2 - 13x - 5$ $(3x + 1)(2x - 5)$
(ii) $(x + y)^2 - 25z^2$ $(x + y + 5z)(x + y - 5z)$	(iv) $27x^3 + 8y^3$ $(3x)^3 + (2y)^3$ $= (3x + 2y)(9x^2 - 6xy + 4y^2)$

2. A quadratic function has roots of -2 and -1. It also contains the point (-3, 8). Evaluate the function in the form $ax^2 + bx + c = 0$ where $a, b, c \in Z$

$f(x) = k[(x+2)(x+1)]$

$f(x) = k[x^2 + 3x + 2]$

$(-3, 8) \Rightarrow f(-3) = 8$

$f(-3) = k[(-3)^2 + 3(-3) + 2] = 8$

$k(2) = 8$

$k = 4$

$\Rightarrow f(x) = 4(x^2 + 3x + 2)$

$f(x) = 4x^2 + 12x + 8$

3. The function $f(x) = 2x^2 + 4x - 4$ can be expressed as $a(x+b)^2 + c$, where $a, b, c \in Z$

(i) Find the values of a, b and c.

(ii) Hence, find the co-ordinates of the local minimum of the curve.

(iii) Solve the equation $f(x) = 0$, writing your answers in surd form.

(iv) Where does the graph cut the y-axis?

(v) Draw a rough sketch of $f(x)$ on the graph paper given.

(i) Using method of completing the square:

$$2x^2 + 4x - 4 = 2(x^2 + 2x - 2)$$

$$= 2(\underbrace{x^2 + 2x + 1} - 1 - 2)$$

$$= 2((x+1)^2 - 3)$$

$$f(x) = 2(x+1)^2 - 6$$

(ii) Minimum: $x = -1$

$f(\ ) = 2(-1+1)^2 - 6 = -6$

Min: $(-1, -6)$

(iii) $f(x) = 0$

$2(x+1)^2 - 6 = 0$

$2(x+1)^2 = 6$

$(x+1)^2 = 3$

$x + 1 = \pm\sqrt{3}$

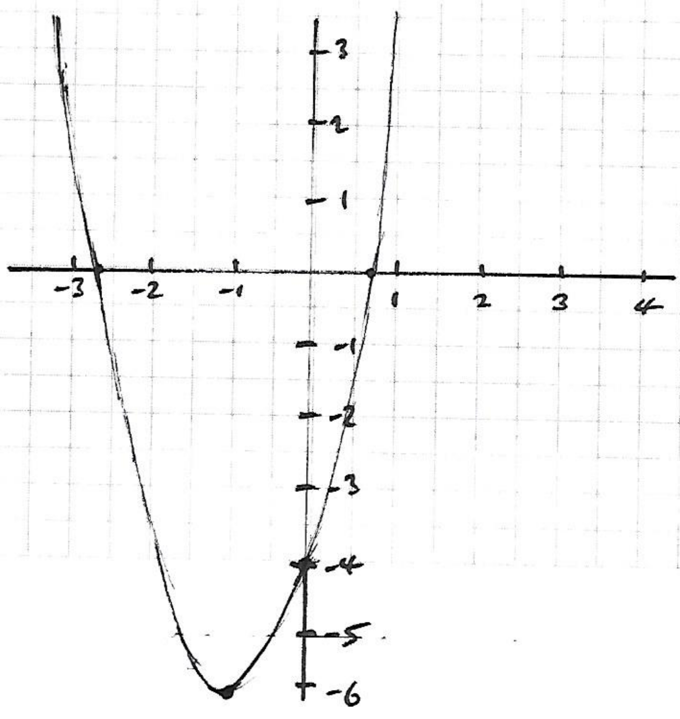
$x = -1 \pm \sqrt{3}$

(iv) At y-axis, $x = 0$

$f(0) = 2(0)^2 + 4(0) - 4$

$f(0) = -4$

$\Rightarrow (0, -4)$



4. Let $f(x)=\dfrac{x^3-1}{x^2-1}$, with $x\neq\pm1$ and $g(x)=\dfrac{x^2+x+1}{x^2-x-2}$, with $x\neq-1,2$.

If $f(x)\div g(x)=ax+b$, find the value of a and b.

$$f(x)\div g(x)=\frac{x^3-1}{x^2-1}\div\frac{x^2+x+1}{x^2-x-2}$$

$$=\frac{x^3-1}{x^2-1}\times\frac{x^2-x-2}{x^2+x+1}$$

$$=\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}\times\frac{(x-2)(x+1)}{x^2+x+1}=x-2$$

$\boxed{a=1,\ b=-2}$

5. Show that $1-x+x^2-\dfrac{1}{1+x}=\dfrac{x^3}{1+x}$ for $x\neq-1$.

$$1-x+x^2-\frac{1}{1+x}=\frac{1-x+x^2}{1}-\frac{1}{1+x}$$

$$=\frac{(1+x)(1-x+x^2)-1}{1+x}$$

$$=\frac{x^3+1-1}{1+x}=\frac{x^3}{1+x}$$

6. Prove that $k+1$ is a root of the function $f(x)=x^2-2x-k^2+1$, where k is a constant.

If $k+1$ is a root $\Rightarrow f(k+1)=0$

$$f(k+1)=(k+1)^2-2(k+1)-k^2+1$$

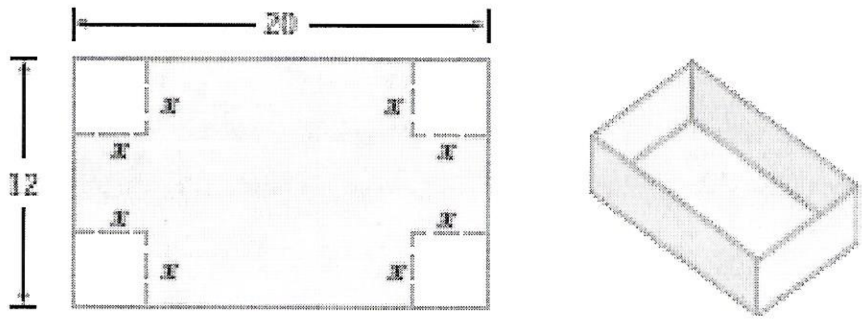
$$=k^2+2k+1-2k-2-k^2+1$$

$$=+2-2$$

$$=0$$

$\Rightarrow$ $k+1$ is a root.

7. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 cm by 20 cm by cutting out equal squares of side at each corner and then folding up the sides as in the figure.



(i) Express the volume of the box as a function of x.

length $= 20-2x$

width $= 12-2x$

height $= x$

$$Vol = (20-2x)(12-2x)(x)$$

$$= x\,(240-40x-24x+4x^2)$$

$$V(x) = 4x^3-64x^2+240x$$

(ii) What is the volume when $x = 2$cm

$$V(x) = (20-2x)(12-2x)(x)$$

$$V(2) = (20-2(2))(12-2(2))(2)$$

$$= (16)(8)(2)$$

$$= 256 \text{ cm}^3$$