TY Honours Maths –Problem Set 9

Name of Student: _____ **Final Mark:** _____

1. Write down a quadratic equation that has roots of ⅓ and ¾ in the form of $ax^2+bx+c=0$ where $a,b,c\in Z$.

$x=\frac{1}{3}$ | $x=\frac{3}{4}$

$3x=1$ | $4x=3$

$3x-1=0$ | $4x-3=0$

$(3x-1)(4x-3)=0$

$12x^2-13x+3=0$

OR

$x^2-(\text{Sum of r's})x+(\text{prod of r's})=0$

2. Solve the following equation leaving your answers in surd form:

$$x^2+8x-2=0$$

$a=1,\ b=8,\ c=-2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-8\pm\sqrt{8^2-4(1)(-2)}}{2(1)}$$

$$x=\frac{-8\pm\sqrt{72}}{2}=\frac{-8\pm\sqrt{36}\sqrt{2}}{2}$$

$$x=\frac{-8\pm6\sqrt{2}}{2}=-4\pm3\sqrt{2}$$

3. A quadratic function has roots of 2 and -3. It also contains the point (1, -12). Evaluate the function.

$f(x)=k[(x-2)(x+3)]$

$f(x)=k(x^2+x-6)$

$(1,-12)\Rightarrow f(1)=-12$

$f(1)=k(1^2+1-6)=-12$

$\Rightarrow -4k=-12$

$k=3$

$f(x)=3(x^2+x-6)$

$=3x^2+3x-18$

4. Solve the following system of equations.

$$y=2x+2$$
$$xy=4$$

$xy=4$

$x(2x+2)=4$

$2x^2+2x=4$

$2x^2+2x-4=0$

$2(x^2+x-2)=0$

$(x+2)(x-1)=0$

$x=-2$ $x=1$

$y=2(-2)+2$ $y=2(\ )+2$

$y=-2$ $y=4$

$(-2,-2)$ $(1,4)$

5. The function $f(x) = 2x^2 + 8x - 2$ can be expressed as $a(x+b)^2 + c$, where $a, b, c \in Z$

(i) Using the method of completing the square, find the values of a, b and c.

(ii) Hence, find the co-ordinates of the local minimum of the curve.

(iii) Solve the equation $f(x) = 0$, writing your answers in surd form.

(i) $f(x) = 2x^2 + 8x - 2 = 2(x^2 + 4x - 1)$

$= 2(x^2 + 4x + 4 - 4 - 1)$

$= 2((x+2)^2 - 5) = 2(x+2)^2 - 10$

$a = 2,\ b = 2,\ c = -10$

(ii) $f(x) = 2(x+2)^2 - 10$ min when $x = -2$

$f(-2) = 2(-2+2)^2 - 10 = -10$

Minimum: $(-2, -10)$

(iii) $f'(x) = 2x^2 + 8x - 2 = 0$

$f'(x) = 2(x+2)^2 - 10 = 0$

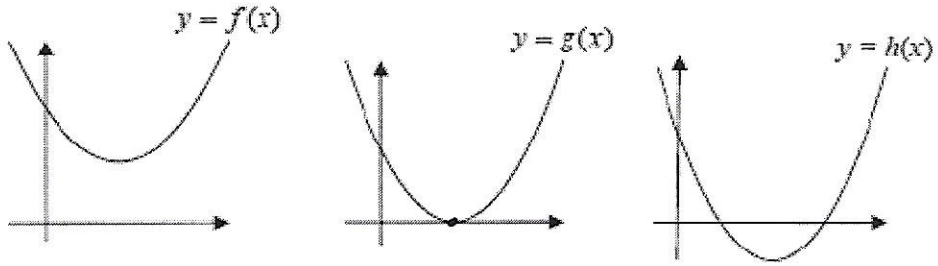
$2(x+2)^2 = 10$

$(x+2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

$x = -2 \pm \sqrt{5}$

6.(i) The graphs of three quadratic functions, f, g and h, are shown.



In each case, state the nature of the roots of the function

Function	Nature of Roots
$f(x)$	Not Real
$g(x)$	Real and Equal
$h(x)$	Real and different.

(ii) By evaluating the discriminant, or otherwise, match each curve above to one of the following functions giving a reason for your choice:

x^2+5x+5 , x^2+2x+5, and x^2-4x+4

$f(x) =$ x^2+2x+5 Reason: $b^2-4ac = 2^2-4(1)(5) = -16 < 0$

Since discriminant $< 0 \Rightarrow f(x)$

$g(x) =$ x^2-4x+4 Reason: $b^2-4ac = (-4)^2-4(1)4 = 0$

Since discriminant $= 0 \Rightarrow g(x)$

$h(x) =$ x^2+5x+5 Reason: $b^2-4ac = 5^2-4(1)(5) = 20 > 0$

Since discriminant $> 0 \Rightarrow h(x)$