Problem Set 10 – Solutions

1 (i) For real roots, $b^2-4ac \geq 0$

$$a = 1+2k, \quad b = -10, \quad c = k-2$$

$$b^2-4ac \geq 0$$

$$(-10)^2-4(1+2k)(k-2) \geq 0$$

$$100-4(k-2+2k^2-4k) \geq 0$$

$$100-4k+8-8k^2+16k \geq 0$$

$$-8k^2+12k+108 \geq 0$$

$$8k^2-12k-108 \geq 0$$

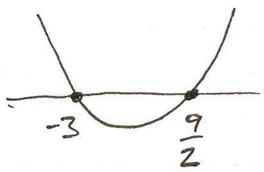
$$2k^2-3k-27 \geq 0$$

$$k \leq -3 \text{ and } k \geq \frac{9}{2}$$

$$2k^2-3k-27=0$$

$$(2k-9)(k+3)=0$$

$$k=\frac{9}{2} \quad | \quad k=-3$$



(ii) $$2\log y = \log 2 + \log x$$

$$\log y^2 = \log 2x$$

$$y^2 = 2x$$

$$2^y = 4^x$$

$$2^y = (2^2)^x$$

$$2^y = 2^{2x}$$

$$y = 2x$$

$$y^2 = y$$

$$y^2 - y = 0$$

$$y(y-1) = 0$$

$y = 0$, $x = 0$ | $y = 1$, $x = \frac{1}{2}$

(iii) $$2^{2x+1} - 15(2^x) = 8$$

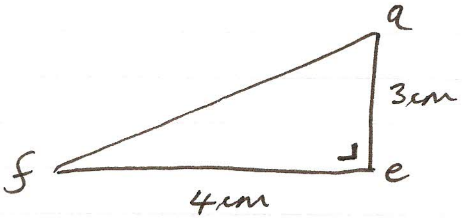
$$2u^2 - 15u - 8 = 0$$

$$(2u+1)(u-8) = 0$$

$u = -\frac{1}{2}$, $2^x = -2^{-1}$, No soln | $u = 8$, $2^x = 2^3$, $x = 3$

$$2^{2x+1}$$
$$= 2^{2x}.2$$
$$= 2.(2^x)^2$$
$$= 2u^2$$

2 (i)



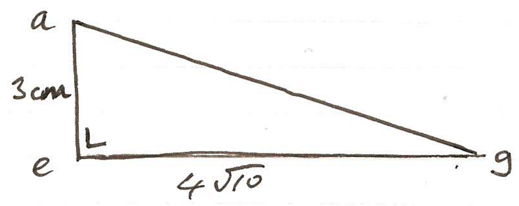
$|af|^2 = |ae|^2 + |ef|^2$

$|af|^2 = 3^2 + 4^2$

$|af|^2 = 25$

$\Rightarrow |af| = 5$

(ii)



$|eg|^2 = |ef|^2 + |fg|^2$

$= 4^2 + 12^2$

$|eg|^2 = 16 + 144 = 160$

$|eg| = \sqrt{160} = 4\sqrt{10}$

$|ag|^2 = |ae|^2 + |eg|^2$

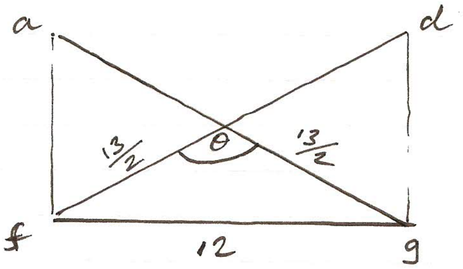
$= 3^2 + (4\sqrt{10})^2$

$= 9 + 160$

$|ag|^2 = 169 \quad \Rightarrow |ag| = \sqrt{169} = 13$

(iii)

$|ag| = |df|$ and diagonals bisect each other.



Find θ

$12^2 = \left(\frac{13}{2}\right)^2 + \left(\frac{13}{2}\right)^2 - 2\left(\frac{13}{2}\right)\left(\frac{13}{2}\right)\cos\theta$

$\cos\theta = -0.7041$

$\theta = \cos^{-1}(-0.7041)$

$\theta = 134.76$

$\Rightarrow$ Acute Angle $= 180 - 134.76$

$= 45.24$

$\approx 45^\circ$

3. $f(x) = a\left[x^2 - (\text{Sum of roots})x + (\text{Prod. of roots})\right]$

$= a\left[x^2 - (-1)x + (-6)\right]$

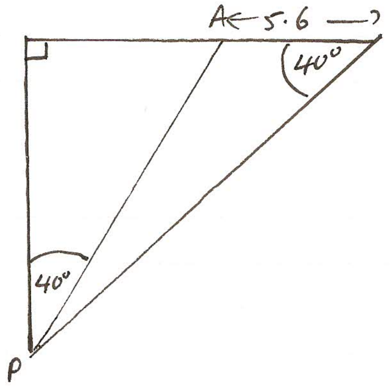
$= a\left(x^2 + x - 6\right)$

$(1, -12) \Rightarrow f(1) = a\left(1^2 + 1 - 6\right) = -12$

$\Rightarrow \quad -4a = -12$

$a = 3$

$f(x) = 3x^2 + 3x - 18$

4. 

$$\frac{|AP|}{\sin 40^\circ} = \frac{5.6}{\sin 10^\circ}$$

$$|AP| = \frac{5.6(\sin 40^\circ)}{\sin 10^\circ}$$

$$|AP| = 20.73$$

$$\cos 40^\circ = \frac{|TP|}{20.73}$$

$$|TP| = 20.73(\cos 40^\circ)$$

$$= 15.8\text{m}$$

5. (i) $z^3 - 1 = (z-1)(z^2 + z + 1)$

(ii) $z^2 + z + 1$

$a = 1, \ b = 1, \ c = 1$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{3}\sqrt{-1}}{2}$$

$$= \frac{-1 \pm \sqrt{3}i}{2}$$

Roots: $z = 1$

$z = -1 + \sqrt{3}i$

$z = -1 - \sqrt{3}i$

6. $y = -2x + 6$

Co-ords of A : $(3,0)$

Co-ords of D : $(0,6)$

- Must get co-ords of P

Equation of [AP] :

$A(3,0)$ slope $= \frac{1}{2}$ $\left([AP] \perp [AD] \right)$

$y - 0 = \frac{1}{2}(x-3)$

$2y = x - 3$

Co-ords of P : $(0, -\frac{3}{2})$

- Finally : $|PD| = |PO| + |OD|$

$= \frac{3}{2} + 6$

$= 7 \cdot 5$