Scoil Mhuire V- Hons Maths: [2014-2015]

Problem Set 9 – For Monday February 23rd.

1. Given that $f(x) = 2^x$ and $f(x+3) - f(x+1) = k\,f(x)$, find the value of k.

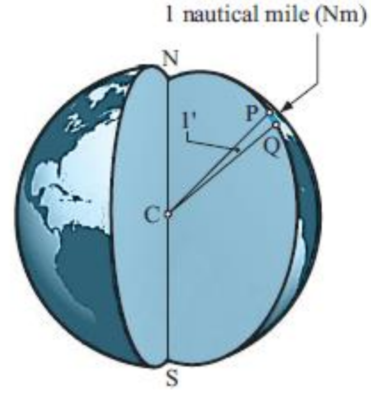
2. Solve the following system of simultaneous equations:

$$\log_2 x - \log_2 2 = \log_2(1-y) \text{ and } \log_2 x + \log_2(x+2y) = 3.$$

3. Express $\sqrt{3}+i$ in polar form and hence show that $\sqrt{3}+i$ is a root of the equation $z^3 + 64 = 0$

4. Write $\frac{1-\sqrt{3}\,i}{2}$ in polar form and hence write $\left(\frac{1-\sqrt{3}\,i}{2}\right)^{13}$ in the form $x + yi$.

5. A **nautical mile** (n mile) is the distance on the Earth's surface that subtends an angle of 1 minute (where 1 minute = 1/60 degree) of the Great Circle arc measured from the centre of the Earth (arc PQ in diagram). A **knot** is a speed of 1 nautical mile per hour.



(a) Given that the radius of the Earth is 6370 km, show that 1 nautical mile is approximately equal to 1.853 km.

(b) Calculate how long it would take a plane to fly from Perth to Adelaide (a distance of 2130 km) if the plane can fly at 480 knots.

6. (i) Copy the following table and fill in the missing values correct to one decimal place.

x	-90°	-60°	-45°	-30°	0°	30°	45°	60°	90°
Tan x									

x	105°	120°	135°	150°	180°	210°	225°	240°	270°
Tan x									

(ii) Draw a graph of $f(x) = Tan\ x$ in the domain $-90° \le x \le 270°$

(iii) What is the period and range of the function.

(iv) Show how the graph can be used to estimate the value of *Tan 405°*. Explain your method.

7. A vegetable gardener has 40 m of fencing to enclose a rectangular garden plot where one side is an existing brick wall. The width of the plot is x m.

(i) Draw a rough diagram of the garden plot, showing both sides in terms of x.
(ii) Show that the area A enclosed is given by $A = -2x^2 + 40x$ m^2.
(iii) Find x such that the vegetable garden has maximum area. [Hint: complete the square]
(iv) What is the maximum area?

Some Answers 1. k = 6 **2.** x = 4, y = -1 **5** (a) Proof (b) 2.4 hours 7. (iii) x = 10m (iv) 200 m^2