Scoil Mhuire V – Hons Maths [2014-15]

Problem Set 8 – For Monday 9th February

1. Let $f(x) = x^2 - 7x + 12$. Show that $\dfrac{f(x)}{f(x+1)}$ simplifies to $\dfrac{x-4}{x-2}$.

2. (a) Using the same axes and scales draw graphs of $f(x) = |x-1|$ and $g(x) = 3$.

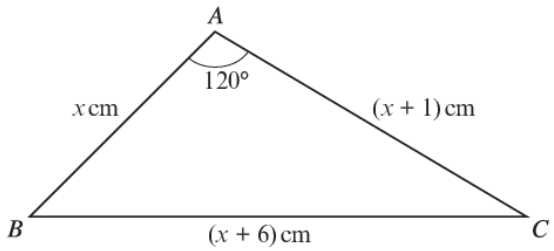
(b) Find from the graph the values of x for which (i) $|x-1| = 3$ and (ii) $|x-1| > 3$.

(c) Verify your answers to part (b) algebraically.

3. The diagram below shows a sketch of the triangle ABC with $|AB| = x$ cm, $|AC| = (x + 1)$ cm, $|BC| = (x + 6)$ cm and $|\angle BAC| = 120°$.

(i) Show that x satisfies the equation: $2x^2 - 9x - 35 = 0$, and hence evaluate x.

(ii) Find the area of triangle ABC. Give your answer correct to two decimal places.



4. Find all values of $\theta \in R$ for which $Sin\ 3\theta = \dfrac{1}{2}$. Give your answer in radians.

5. (i) Using the same axes and scales, sketch the following functions:

$f(x) = \cos x$ and $g(x) = \sin 2x$ for $0 \le x \le 2\pi$.

(ii) Write down the period and range of each function.

(iii) Estimate from the graphs in parts (i) the solutions to the following equation:

$\cos x - \sin 2x = 0$ for $0 \le x \le 2\pi$

6. Solve, using trial and error, the cubic equation $x^3 - x^2 - 14x + 24 = 0$

7. A stone is thrown into the air and its height in metres above the ground is given by the function

$h(t) = -5t^2 + 30t + 2$ where t is the time (in seconds) from when the stone is thrown.

(a) How high above the ground is the stone at time t = 3 seconds?
(b) How high above the ground was the stone released?
(c) At what times was the stone's height above the ground 27 m?

8. Given that $z = 2 - 3i$, find the value of $p, q \in Q$ in the following equation: $z + i + 3(p + 2qi) = iz - 5$.

9. Find two complex numbers in the form of $a + bi$ such that $(a + bi)^2 = 5 + 12i$, where $a, b \in R$.

10. For all complex numbers $z = a + bi$ prove the following identities: (i) $z.\overline{z} = |z|^2$ (ii) $z + \overline{z} = 2\,\mathrm{Re}(z)$

Answers: **1.** ------- **2.** (b) Estimate (c) (i) x = 4,-2 (ii) $x > 3$ and $x < -2$ **3.** (i) x = 7 (ii) 24.25 cm^2 **4.** --------
5 (ii) f(x): range =[-1,1] and Period is 2π . g(x): range =[-1,1] and period is π (iii) $x = 30°, 90°, 150°$ and $270°$
6. Roots x = - 4, 2, 3 **7.** (a) 47m (b) 2m (c) $t = 5s$ and $t = 1s$ **8.** $\frac{-4}{3}, \frac{2}{3}$ **9.** $3 + 4i$ and $-3 - 4i$ **10.** -------