Scoil Mhuire V (14-15) – Problem Set 4 (Solutions)

1 (i) $4x^2+14x+10$
$= (2x+5)(2x+2)$

(ii) $3x^2-21x-24$
$= (3x+3)(x-8)$

(iii) $5p^2+4pq-q^2$
$= (5p-q)(p+q)$

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2 Roots not real $\Rightarrow b^2-4ac<0$

$a=1+2k,\quad b=-10,\quad c=k-2$

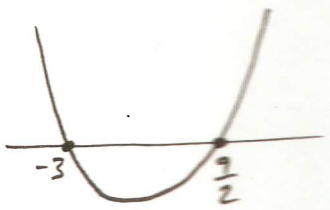
$(-10)^2-4(1+2k)(k-2)<0$

$100-4(k-2+2k^2-4k)<0$

$100+12k+8-8k^2<0$

$8k^2-12k-108<0$

$-3<x<\frac{9}{2}$



$8k^2-12k-108=0$

$2k^2-3k-27=0$

$(2k-9)(k+3)=0$

$k=\frac{9}{2}$ | $k=-3$

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3 (i) $\log_3(10x-7)-\log_3(x+1)=2$

$\log_3\left(\frac{10x-7}{x+1}\right)=2$

$\frac{10x-7}{x+1}=3^2$

$\frac{10x+7}{x+1}=9$

$10x+7=9x+9$

$x=2$

(ii) $\ln(4x+1)=2.5649$

$e^{\ln(4x+1)}=e^{2.5649}$

$4x+1=12.999$

$4x=12$

$x=3$

(iii) $4^x=8$

$(2^2)^x=2^3$

$2^{2x}=2^3$

$2x=3$

$x=\frac{3}{2}$

(iv) $e^{2x+4}=0.1353$

$\ln(e^{2x+4})=\ln(0.1353)$

$2x+4=-2$

$2x=-6$

$x=-3$

4// If $x+3$ is a factor

$\Rightarrow$ $x=-3$ is a root

$\Rightarrow$ $f(-3)=0$

$$f(-3)=(-3)^3+k(-3)^2-4(-3)-12=0$$

$$-27+9k+12-12=0$$

$$9k=27$$

$$k=3$$

6// $3^{x+1}+3^{1-x}=10$

$$3^x.3^1+3.3^{-x}=10$$

$$3.3^x+\frac{3}{3^x}=10 \quad \text{let } y=3^x$$

$$3y+\frac{3}{y}=10$$

$$3y^2+3=10y$$

$$3y^2-10y+3=0$$

$$(3y-1)(y-3)=0$$

$y=\frac{1}{3}$	$y=3$
$3^x=3^{-1}$	$3^x=3^1$
$x=-1$	$x=1$

$x=-1, 1$

5// $\frac{x+1}{x-1}<4$

$$\frac{x+1}{x-1}(x-1)^2<4(x-1)^2$$

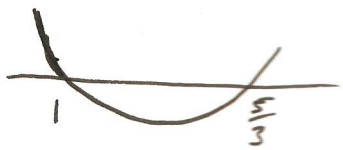
$$(x+1)(x-1)<4(x^2-2x+1)$$

$$x^2-1<4x^2-8x+4$$

$$3x^2-8x+5>0$$

$$(3x-5)(x-1)=0$$

$x=\frac{5}{3}$ | $x=1$



$x<1$ and $x>\frac{5}{3}$

7// Trial + Error

$$f(-3)=4(-3)^3+10(-3)^2-7(-3)-3=0$$

$$-108+90+21-3=0$$

$$-111+111=0$$

$$0=0 \ \checkmark$$

$$x+3 \overline{)\,4x^3+10x^2-7x-3}$$

$$4x^2-2x-1$$

Now, Solve $4x^2-2x-1=0$

$$x=\frac{2\pm\sqrt{4+16}}{2(4)}=\frac{2\pm2\sqrt{5}}{8}=\frac{1\pm\sqrt{5}}{4}$$

$x=\frac{1+\sqrt{5}}{4}$ $\quad$ $x=\frac{1-\sqrt{5}}{4}$

8 // $x = -1, x = 4, x = -2$

Factors $x+1, x-4, x+2$

$$p(x) = (x+1)(x-4)(x+2)$$
$$= (x+1)(x^2 - 2x - 8)$$
$$= x^3 - 2x^2 - 8x + x^2 - 2x - 8$$
$$= x^3 - x^2 - 10x - 8$$

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9 //

(i) $f(x) = x^2 + 6x + 2$

$= \underbrace{x^2 + 6x + 9} - 9 + 2$

$f(x) = (x+3)^2 - 7$

$\Rightarrow a = 3, b = -7$

(ii) minimum occurs when $x = -3$

$f(-3) = (-3+3)^2 - 7$

$= 0 - 7$

$= -7$

(iii) $f(x) = 0$

$\Rightarrow (x+3)^2 - 7 = 0$

$(x+3)^2 = 7$

$x + 3 = \pm\sqrt{7}$

$x = -3 \pm \sqrt{7}$

10 //

(i) $T = 400e^{-0.05t} + 25$

when $t = 0$, $T = ?$

$T = 400e^{-0.05(0)} + 25$

$= 400(1) + 25$

$= 425$

(ii) $T = 300$, $t = ?$

$T = 400e^{-0.05t} + 25$

$300 = 400e^{-0.05t} + 25$

$275 = 400e^{-0.05t}$

$\frac{275}{400} = e^{-0.05t}$

$0.6875 = e^{-0.05t}$

$\ln(0.6875) = \ln(e^{-0.05t})$

$-0.3747 = -0.05t$

$t = 7.49$ mins

(iii) $T = 400e^{-0.05t} + 25$

as $t \to \infty$, $e^{-0.05t} \to 0$

$\Rightarrow T = 400(0) + 25$

$T = 25^\circ C$

when t gets very large, T won't go below $25^\circ C$