Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2008

MARKING SCHEME

MATHEMATICS

HIGHER LEVEL

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LEAVING CERTIFICATE EXAMINATION 2008

MATHEMATICS – HIGHER LEVEL – PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work of merit is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

1. **(a)** Simplify fully $\frac{x^2+4}{x^2-4}-\frac{x}{x+2}$.

Correct Numerator	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

1 (a)

$$\frac{x^2+4}{x^2-4}-\frac{x}{x+2}=\frac{x^2+4}{(x-2)(x+2)}-\frac{x}{(x+2)}$$

$$=\frac{(x^2+4)-x(x-2)}{(x-2)(x+2)}$$

$$=\frac{x^2+4-x^2+2x}{(x-2)(x+2)}$$

$$=\frac{2x+4}{(x-2)(x+2)}$$

$$=\frac{2(x+2)}{(x-2)(x+2)}=\frac{2}{x-2}$$

Blunders (-3)

B1 Factors once only

B2 Indices

B3 Incorrect cancellation

Part (b)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

1. (b)

(b) Given that one of the roots is an integer, solve the equation

$$6x^3 - 29x^2 + 36x - 9 = 0.$$

Getting (*x* – 3) as factor	**5 marks**	**Att 2**
Division	**5 marks**	**Att 2**
Remaining two factors	**5 marks**	**Att 2**
Roots	**5 marks**	**Att 2**

1. (b)

$$f(x) = 6x^3 - 29x^2 + 36x - 9$$

$$f(1) = 6 - 29 + 36 - 9 \neq 0$$

$$f(2) = 48 - 116 + 72 - 9 \neq 0$$

$$f(3) = 162 - 261 + 108 - 9 = 270 - 270 = 0.$$

$\therefore\ x = 3 \Rightarrow (x - 3)$ is a factor.

$$(x-3)(6x^2 + ax + 3) = 6x^3 - 29x^2 + 36x - 9.$$

$$\therefore\ a - 18 = -29 \ \Rightarrow\ a = -11.$$

$$\therefore\ 6x^2 - 11x + 3 = 0 \ \Rightarrow\ (3x-1)(2x-3) = 0.$$

$$\therefore\ 3x - 1 = 0 \text{ or } 2x - 3 = 0 \ \Rightarrow\ x = \frac{1}{3} \text{ or } x = \frac{3}{2}.$$

Roots are $3, \frac{1}{3}, \frac{3}{2}$.

OR

Getting $(x-3)$ as factor	**5 marks**	**Att 2**
Division	**5 marks**	**Att 2**
Remaining two factors	**5 marks**	**Att 2**
Roots	**5 marks**	**Att 2**

1. (b)

$$f(x)=6x^3-29x^2+36x-9$$

$$f(1)=6-29+36-9\neq 0$$

$$f(-1)\qquad\qquad\neq 0$$

$$f(3)=6(27)-29(9)+36(3)-9$$

$$=162-261+108-9$$

$$=270-270$$

$$f(3)=0\qquad\qquad\Rightarrow (x-3)\text{ is a factor}$$

$$\begin{array}{r} 6x^2-11x+3 \\ x-3\overline{)6x^3-29x^2+36x-9} \\ \underline{6x^3-18x^2} \\ -11x^2+36x \\ \underline{-11x^2+33x} \\ 3x-9 \\ \underline{3x-9} \end{array}$$

$$f(x)=(x-3)(6x^2-11x+3)$$

$$=(x-3)[(3x-1)(2x-3)]$$

$$f(x)=0\Rightarrow(x-3)(3x-1)(2x-3)=0$$

$$\Rightarrow x=3,\tfrac{1}{3},\tfrac{3}{2}$$

Blunders (-3)

B1 Test for root
B2 Deduction of factor from root or no deduction
B3 Indices
B4 Root formula (once only)
B5 Deduction of root from factor or no deduction
B6 Not like to like when equating coefficients

Slips (-1)

S1 Numerical
S2 Not changing sign when subtracting in division

Worthless

W1 $x(6x^2-29x+36)=9$, with or without further work

NOTE If there is a remainder after division, or incomplete division, candidates can only get Att at most for remaining factors and roots.

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

1. (c)

(c) Two of the roots of the equation $ax^3 + bx^2 + cx + d = 0$ are p and $-p$.
Show that $bc = ad$.

($x^2 - p^2$) a factor	**5 marks**	**Att 2**
Divison	**5 marks**	**Att 2**
Remainder= 0	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

1. (c)

$$x = p \text{ and } x = -p \Rightarrow (x-p)(x+p) = x^2 - p^2 \text{ is a factor.}$$

$$ax^3 + bx^2 + cx + d = \left(x^2 - p^2\right)\left(ax - \frac{d}{p^2}\right).$$

$$\therefore\ b = -\frac{d}{p^2} \text{ and } c = -ap^2.$$

$$p^2 = -\frac{c}{a} \Rightarrow b = \frac{ad}{c} \Rightarrow bc = ad.$$

OR

$(x^2 - p^2)$ a factor	**5 marks**	**Att 2**
Linear Factor	**5 marks**	**Att 2**
Equating Coefficient	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

1 (c)

p and $(-p)$ are roots $\Rightarrow (x-p)$ and $(x+p)$ are factors

$\Rightarrow \left(x^2-p^2\right)$ is a factor

$$
\begin{array}{r}
ax+b \\
x^2-p^2\,\overline{\big)\,ax^3+bx^2+cx+d} \\
\underline{ax^3-ap^2x} \\
bx^2+\left(c+ap^2\right)x \\
\underline{bx^2-bp^2} \\
\left(c+ap^2\right)x+bp^2+d
\end{array}
$$

Since $\left(x^2-p^2\right)$ is factor, remainder = 0

$$\left(c+ap^2\right)x+\left(bp^2+d\right)=(0)x+(0)$$

$$\Rightarrow (i): c+ap^2=0$$

$$p^2=-\frac{c}{a}$$

(ii) $bp^2+d=0$

$$p^2=-\frac{d}{b}$$

From (i) and (ii): $-\frac{c}{a}=-\frac{d}{b}$

$$cb=ad$$

OR

Two Equations	**5 marks**	**Att 2**
Adding	**5 marks**	**Att 2**
Subtracting	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

1 (c)

Since p is a root of $ax^3 + bx^2 + cx + d = 0$

then $a(p)^3 + b(p)^2 + c(p) + d = 0$

$$ap^3 + bp^2 + cp + d = 0 \text{.................(i)}$$

Similarly $(-p)$ is a root

$$a(-p)^3 + b(-p)^2 + c(-p) + d = 0$$

$$-ap^3 + bp^2 - cp + d = 0 \text{................(ii)}$$

Adding (i) and (ii): $2bp^2 + 2d = 0$

$$bp^2 = -d$$

$$p^2 = -\frac{d}{b} \text{..................(iii)}$$

Subtracting (i) and (ii): $2ap^3 + 2cp = 0$

$$ap^2 = -c$$

$$p^2 = -\frac{c}{a} \text{..................(iv)}$$

From (iii) and (iv): $-\frac{d}{b} = -\frac{c}{a}$

$$ad = bc$$

Blunders (-3)

B1 Indices

B2 Factor not $(x^2 - p^2)$ once only

B3 Not like to like when equating coefficients

Slips (-1)

S1 Not changing sign when subtracting in division

Attempts

A1 Any effort at division

A2 Other factor not linear – cannot now get any more marks

QUESTION 2

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

2. (a)

(a) Express $x^2 + 10x + 32$ in the form $(x+a)^2 + b$.

Split	**5 marks**	**Att 2**
Express	**5 marks**	**Att 2**

2. (a)

$$x^2 + 10x + 32 = x^2 + 10x + 25 + 7 = (x+5)^2 + 7.$$

* Accept solutions based on two values of x

OR

Equating Coefficients	**5 marks**	**Att 2**
Solving equations	**5 marks**	**Att 2**

2. (a)

$$x^2 + 10x + 32 = (x+a)^2 + b$$

$$x^2 + (10)x + 32 = x^2 + (2a)x + (a^2 + b)$$

Equating Coefficients (i) $10 = 2a$

$5 = a$

(ii) $a^2 + b = 32$

$25 + b = 32$

$b = 7$

Blunders (-3)

B1 Indices

B2 Expansion of $(x+a)^2$ once only

B3 Completing square

B4 Not like to like when equating coefficients

B5 No ‘a’ or no deduction ‘a’

B6 No ‘b’ or no deduction ‘b’

Slips (-1)

S1 Numerical

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

2. (b)

(b) α and β are the roots of the equation $x^2 - 7x + 1 = 0$.

(i) Find the value of $\alpha^2 + \beta^2$.

(ii) Find the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$.

(i) Values: $\alpha+\beta$ & $\alpha\beta$, or solve quad.	**5 marks**	**Att 2**
$\alpha^2+\beta^2$	**5 marks**	**Att 2**
(ii) Factors	**5 marks**	**Att 2**
Value	**5 marks**	**Att 2**

2. (b) (i)

$$\alpha + \beta = -\frac{b}{a} = 7 \text{ and } \alpha\beta = \frac{c}{a} = 1.$$

$$(\alpha + \beta)^2 = 49 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 49.$$

$$\therefore \alpha^2 + \beta^2 = 47.$$

2. (b) (ii)

$$\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{\alpha^3\beta^3} = \frac{(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)}{1}$$

$$= 7(47 - 1) = 322.$$

Blunders (-3)

B1 Indices

B2 Incorrect sum

B3 Incorrect product

B4 Statement incorrect

B5 Factors

Slips (-1)

S1 Numerical

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

2. (c)

(c) Show that if *a* and *b* are non-zero real numbers, then the value of $\frac{a}{b}+\frac{b}{a}$ can never lie between -2 and 2.

Hint : consider the case where *a* and *b* have the same sign separately from the case where *a* and *b* have opposite sign.

Inequality for same sign	**5 marks**	**Att 2**
Deduction from above	**5 marks**	**Att 2**
Inequality for opposite sign	**5 marks**	**Att 2**
Deduction from above	**5 marks**	**Att 2**

Case 1: *a* and *b* have same sign.

In this case, $\frac{a}{b}+\frac{b}{a}>0$, so we need to show that $\frac{a}{b}+\frac{b}{a}>2$.

$$\frac{a}{b}+\frac{b}{a}>2$$
$$\Leftrightarrow a^2+b^2>2ab, \quad (\text{since } ab>0)$$
$$\Leftrightarrow a^2-2ab+b^2>0$$
$$\Leftrightarrow (a-b)^2>0 \qquad \text{True.}$$

Case 2: *a* and *b* have opposite sign.

In this case, $\frac{a}{b}+\frac{b}{a}<0$, so we need to show that $\frac{a}{b}+\frac{b}{a}<-2$.

$$\frac{a}{b}+\frac{b}{a}<-2$$
$$\Leftrightarrow a^2+b^2>-2ab, \quad (\text{since } ab<0)$$
$$\Leftrightarrow a^2+2ab+b^2>0$$
$$\Leftrightarrow (a+b)^2>0 \qquad \text{True.}$$

Or

$x+\frac{1}{x}=k$	**5 marks**	**Att 2**
Quadratic	**5 marks**	**Att 2**
b^2-4ac	**5 marks**	**Att 2**
Deduction	**5 marks**	**Att 2**

2.(c)

Let $\frac{a}{b}=x$. Then must show $\left(x+\frac{1}{x}\right)$ is never $\in[-2,2]$

Let $\left(x+\frac{1}{x}\right)=k$. So, need to show $|k|>2$

$$x+\frac{1}{x}=k$$
$$x^2+1=kx$$
$$x^2-kx+1=0$$

For real x, $b^2-4ac>0$

i.e. $k^2-4>0$

$k^2>4$

i.e., $|k|>2$

OR

Mod Value	**5 marks**	**Att 2**
Squaring	**5 marks**	**Att 2**
$\left(\frac{a}{b}-\frac{b}{a}\right)^2$	**5 marks**	**Att 2**
Deduction	**5 marks**	**Att 2**

2.(c)

Need to show. $\left|\frac{a}{b}+\frac{b}{a}\right|>2$

Proof: $\Leftrightarrow\left(\frac{a}{b}+\frac{b}{a}\right)^2>4$

$$\Leftrightarrow\frac{a^2}{b^2}+\frac{b^2}{a^2}+2>4$$
$$\Leftrightarrow\frac{a^2}{b^2}+\frac{b^2}{a^2}-2>0$$
$$\Leftrightarrow\left(\frac{a}{b}-\frac{b}{a}\right)^2>0 \quad \text{True}$$

Blunders (-3)

B1 Inequality sign
B2 Factors
B3 Incorrect deduction or no deduction

QUESTION 3

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

3. (a) Let A be the matrix $\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$.

Find the matrix B, such that $AB = \begin{pmatrix} 4 & 6 \\ 3 & 2 \end{pmatrix}$.

A^{-1}	**5 marks**	**Att 2**
B	**5 marks**	**Att 2**

3 (a) $A.B = \begin{pmatrix} 4 & 6 \\ 3 & 2 \end{pmatrix} \quad \Rightarrow \quad B = A^{-1}\begin{pmatrix} 4 & 6 \\ 3 & 2 \end{pmatrix}$

$$A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{6-5}\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$$

$$B = A^{-1}\begin{pmatrix} 4 & 6 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 6 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -7 & 2 \\ 5 & 0 \end{pmatrix}$$

OR

Four equations	**5 marks**	**Att 2**
Four values	**5 marks**	**Att 2**

3 (a) Let $B = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ Then $AB = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 3 & 2 \end{pmatrix}$

(i): $3p + 5r = 4$ (ii): $3q + 5s = 6$
(iii): $p + 2r = 3$ (iv): $q + 2s = 2$

(i) and (iii):
$$\begin{aligned} 3p + 5r = 4 &\Rightarrow 3p + 5r = 4 \\ p + 2r = 3 &\Rightarrow \underline{3p + 6r = 9} \\ & \quad -r = -5 \quad \Rightarrow \quad r = 5 \quad \Rightarrow \quad p = -7 \end{aligned}$$

(ii) and (iv)
$$\begin{aligned} 3q + 5s = 6 &\Rightarrow 3q + 5s = 6 \\ q + 2s = 2 &\Rightarrow \underline{3q + 6s = 6} \\ & \quad -s = 0 \quad \Rightarrow \quad s = 0 \quad \Rightarrow \quad q = 2 \end{aligned}$$

$$B = \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} -7 & 2 \\ 5 & 0 \end{pmatrix}$$

Blunders (-3)
B1 Formula for inverse
B2 Matrix multiplication

Slips (-1)
S1 Each incorrect element
S2 Numerical

Part (b) 20 (5, 5, 5, 5) marks Att (2, 2, 2, 2)

3. (b) (i) Let $z = \frac{5}{2+i} - 1$, where $i^2 = -1$.

Express z in the form $a + bi$ and plot it on an Argand diagram.

(ii) Use De Moivre's theorem to evaluate z^6.

(i) *z* when multiplied by conjugate	**5 marks**	**Att 2**
Plot	**5 marks**	**Att 2**
(ii) *z* in polar form	**5 marks**	**Att 2**
Value	**5 marks**	**Att 2**

3 (b) (i)

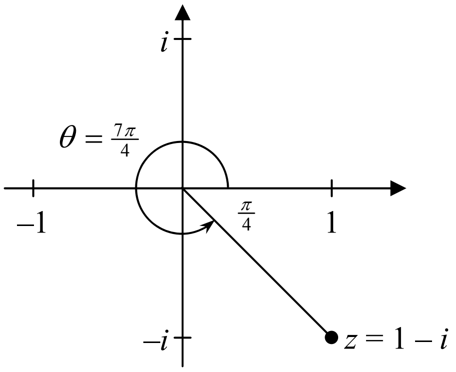
$$z = \frac{5}{2+i} - 1 = \frac{5-(2+i)}{2+i} \quad = \frac{3-i}{2+i}$$

$$= \frac{3-i}{2+i} \cdot \frac{2-i}{2-i}$$

$$= \frac{6-5i+i^2}{4-i^2}$$

$$= \frac{5-5i}{5}$$

$$\Rightarrow z = 1 - i$$



3 (b) (ii)

$$z = r(\cos\theta + i\sin\theta)$$

$$z = 2^{\frac{1}{2}}\left(\cos\tfrac{7\pi}{4} + i\sin\tfrac{7\pi}{4}\right)$$

$$z^6 = \left[2^{\frac{1}{2}}\left(\cos\tfrac{7\pi}{4} + i\sin\tfrac{7\pi}{4}\right)\right]^6$$

$$= (2)^3\left[\cos\tfrac{21\pi}{2} + i\sin\tfrac{21\pi}{2}\right]$$

$$= 8\left[\cos\tfrac{\pi}{2} + i\sin\tfrac{\pi}{2}\right]$$

$$= 8[0 + i]$$

$$= 8i$$

$$\theta = \tfrac{7\pi}{4}$$

$$|z| = r = \sqrt{1 + (-1)^2}$$

$$r = \sqrt{2} = 2^{\frac{1}{2}}$$

Blunders (-3)

B1 Indices
B2 i
B3 $(2+i)(2-i) \neq 5$
B4 Argument
B5 Modulus
B6 Trig Definition
B7 Statement De Moivre once only
B8 Application De Moivre
B9 No plot z or incorrect plot z

Slips (-1)

S1 Trig value

Worthless

W1 Not De Moivre

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

3 (c) Prove, by induction, that

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta \text{ for } n \in \mathbf{N}.$$

***P*(1) or *P*(0)**	**5 marks**	**Att 2**
***P*(*k*)**	**5 marks**	**Att 2**
P(*k* + 1)	**5 marks**	**Att 2**
Proof	**5 marks**	**Att 2**

3 (c)

Prove: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta, \quad [n \in \mathbf{N}]$

Test $n = 0 : (\cos\theta + i\sin\theta)^0 = \cos(0)\theta + i\sin(0)\theta$

$$1 = \cos 0 + i\sin 0$$
$$1 = 1$$

True for $n = 0$

Assume true for $n = k$: $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$

To prove: $(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta$

$$\begin{aligned}(\cos\theta + i\sin\theta)^{k+1} &= (\cos\theta + i\sin\theta)^k(\cos\theta + i\sin\theta)^1\\ &= (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta) \qquad \text{(by ind. hyp.)}\\ &= \cos(k+1)\theta + i\sin(k+1)\theta\end{aligned}$$

So, {true for $n = k \quad \Rightarrow \quad$ true for $n = k+1$}

$\therefore$ True for all $n \in \mathbf{N}$.

Blunders (-3)

B1 Indices
B2 Trig Formula
B3 *i*
B4 Statement De Moivre

* NOTE: Accept $n = 0$ or $n = 1$ for first step

QUESTION 4

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

4. (a) $2+\frac{2}{3}+\frac{2}{9}+........$ is a geometric series.

Find the sum to infinity of the series.

Correct substitution into formula	**5 marks**	**Att 2**
Sum	**5 marks**	**Att 2**

4 (a)

$$S_\infty = \frac{a}{1-r} = \frac{2}{1-\frac{1}{3}} = 3.$$

Blunders (-3)

B1 Formula sum to infinity

B2 Indices

B3 Incorrect ‘*a*’

B4 Incorrect ‘*r*’

Worthless

W1 Uses A.P.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

4 (b) Given that $u_n = 2\left(-\frac{1}{2}\right)^n - 2$ for all $n \in \mathbf{N}$,

(i) write down u_{n+1} and u_{n+2}

(ii) show that $2u_{n+2} - u_{n+1} - u_n = 0$.

(i) Write down	**5 marks**	**Att 2**
(ii) Terms simplified	**5 marks**	**Att 2**
Correct substitution	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

4 (b) (i)

$$u_{n+1} = 2\left(-\frac{1}{2}\right)^{n+1} - 2. \qquad u_{\mathrm{n}+2} = 2\left(-\frac{1}{2}\right)^{n+2} - 2.$$

(ii)

$$2u_{n+2} - u_{n+1} - u_n = 4\left(-\frac{1}{2}\right)^{n+2} - 4 - 2\left(-\frac{1}{2}\right)^{n+1} + 2 - 2\left(-\frac{1}{2}\right)^n + 2.$$

$$= 4\left(\frac{1}{4}\right)\left(-\frac{1}{2}\right)^n - 2\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^n - 2\left(-\frac{1}{2}\right)^n$$

$$= \left(-\frac{1}{2}\right)^n + \left(-\frac{1}{2}\right)^n - 2\left(\frac{-1}{2}\right)^n = 0.$$

Blunders (-3)

B1 Indices

Attempts

A1 Must do some correct relevant work with indices

NOTE:Simplification and substitution can be in any order

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

4 (c) (i) Write down an expression in n for the sum $1+2+3+.........+n$ and an expression in n for the sum $1^2+2^2+3^2+.........+n^2$.

(ii) Find, in terms of n, the sum $\sum_{r=1}^{n}\left(6r^2+2r+5+2^r\right)$.

(i) Formulae	**5 marks**	**Att 2**
(ii) 1st two terms	**5 marks**	**Att 2**
5*n*	**5 marks**	**Att 2**
G.P.	**5 marks**	**Att 2**

4 (c) (i)

$$1+2+3+.........+n=\sum_{1}^{n}n=\frac{n}{2}(n+1).$$

$$1^2+2^2+3^2+.......+n^2=\sum_{1}^{n}n^2=\frac{n}{6}(n+1)(2n+1).$$

(ii)

$$\sum_{r=1}^{n}\left(6r^2+2r+5+2^r\right)=6\sum_{r=1}^{n}r^2+2\sum_{r=1}^{n}r+\sum_{r=1}^{n}5+\sum_{r=1}^{n}2^r$$

$$=n(n+1)(2n+1)+n(n+1)+5n+\frac{2(2^n-1)}{2-1}.$$

$$=n(n+1)(2n+1)+n(n+1)+5n+2(2^n-1)$$

Blunders (-3)

B1 Indices

B2 Incorrect $\sum n$

B3 Incorrect $\sum n^2$

B4 $5n$ term

B5 Formula G.S

B6 Incorrect 'a'

B7 Incorrect r

Slips (-1)

S1 Numerical

QUESTION 5

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

5. (a) Find the range of values of x that satisfy the inequality

$$x^2 - 3x - 10 \le 0.$$

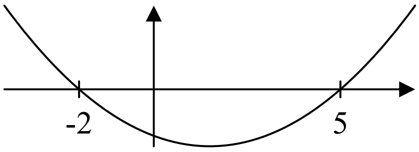
Factors	**5 marks**	**Att 2**
Range	**5 marks**	**Att 2**

5 (a)

$x^2 - 3x - 10 \le 0 \;\Rightarrow\; (x-5)(x+2) \le 0.$

Graph : $x \to (\text{x - 5})(x+2)$

$f(x) \le 0$ when $-2 \le x \le 5$



OR

Factors	**5 marks**	**Att 2**
Range	**5 marks**	**Att 2**

5 (a) $x^2 - 3x - 10 \le 0$

$(x-5)(x+2) \le 0$

Either: I: $x - 5 \ge 0$ and $x + 2 \le 0$

$x \ge 5$ and $x \le -2$

Not Possible

or: II: $x - 5 \le 0$ and $x + 2 \ge 0$

$x \le 5$ and $x \ge -2$

$\therefore$ answer is $-2 \le x \le 5$

Blunders (-3)

B1 Factors
B2 Root from factor
B3 Upper Limit
B4 Lower Limit
B5 Inequality sign
B6 Root formula, once only
B7 Incorrect range
B8 Answer not stated

Slips (-1)

S1 Numerical

Attempts

A1 One inequality sign
A2 Inequality signs ignored

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

5 (b) **(i)** Solve the equation

$$2^{x^2} = 8^{2x+9}.$$

(ii) Solve the equation

$$\log_e(2x+3) + \log_e(x-2) = 2\log_e(x+4).$$

(b)(i) Quadratic	**5 marks**	**Att 2**
Solve	**5 marks**	**Att 2**

5 (b) (i)

$$2^{x^2} = 8^{2x+9} \Rightarrow 2^{x^2} = 2^{6x+27}.$$

$$\therefore\ x^2 - 6x - 27 = 0 \Rightarrow (x-9)(x+3) = 0.$$

$$\therefore\ x = 9 \quad \text{or} \quad x = -3.$$

Blunders (-3)

B1	Indices
B2	Factors
B3	Root formula, once only
B4	Deduction root from factor

(b)(ii) Correct working with logs	**5 marks**	**Att 2**
Correct value *x*	**5 marks**	**Att 2**

5 (b) (ii)

$$\log_e(2x+3) + \log_e(x-2) = 2\log_e(x+4)$$

$$\therefore\ \log_e(2x+3)(x-2) = \log_e(x+4)^2.$$

$$\therefore\ 2x^2 - x - 6 = x^2 + 8x + 16.$$

$$\therefore\ x - 9x - 22 = 0 \Rightarrow (x-11)(x+2) = 0.$$

$$\therefore\ x = 11.\ \ x = -2.$$

Test: $x = 11$ L.H.S.: $\ln(25) + \ln(9) = \ln 225$

R.H.S.: $2\ln(15) = \ln 225$

Test: $x = -2$ L.H.S.: $\ln(-1) + \ln(-4)$, which do not exist

$\therefore$ the only solution is $x = 11$.

Blunders (-3)

B1	Logs
B2	Indices
B3	Factors
B4	Root Formula
B5	Deduction root from factor or no deduction
B6	Excess value

Worthless

W1	Drops ‘Log’

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

5 (c) Show that there are no natural numbers n and r for which $\binom{n}{r-1}, \binom{n}{r}$ and $\binom{n}{r+1}$ are consecutive terms in a geometric sequence.

Definition of G.S.	**5 marks**	**Att 2**
Factorial values inserted	**5 marks**	**Att 2**
Simplified fractions	**5 marks**	**Att 2**
Not natural no	**5 marks**	**Att 2**

5 (c)

If a geometric sequence, then $\dfrac{\binom{n}{r}}{\binom{n}{r-1}} = \dfrac{\binom{n}{r+1}}{\binom{n}{r}}$.

$$\therefore \quad \frac{\dfrac{n!}{r!(n-r)!}}{\dfrac{n!}{(r-1)!(n-r+1)!}} = \frac{\dfrac{n!}{(r+1)!(n-r-1)!}}{\dfrac{n!}{r!(n-r)!}}.$$

$$\therefore \quad \frac{n-r+1}{r} = \frac{n-r}{r+1} \Rightarrow (n-r+1)(r+1) = r(n-r).$$

$\therefore \; nr + n - r^2 - r + r + 1 = nr - r^2 \;\Rightarrow\; n = -1$, which is not a natural number.

Blunders (-3)

B1 Definition of G.S.

B2 Incorrect $\binom{n}{r}$

B3 Incorrect $\binom{n}{r-1}$

B4 Incorrect $\binom{n}{r+1}$

B5 Factorial

B6 Indices

B7 Cross multiplication

B8 Incorrect deduction or no deduction

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a)	**10 marks**	**Att 3**

6. (a) Differentiate $\sqrt{x^3}$ with respect to x.

Part (a)	**10 marks**	**Att 3**

6 (a)

$$f(x) = x^{\frac{3}{2}} \quad \Rightarrow \quad f'(x) = \frac{3}{2}x^{\frac{1}{2}}.$$

Blunders (-3)

B1 Blunder indices

B2 Blunder differentiation

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

6 (b) Let $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

Show that $\dfrac{dy}{dx} = \dfrac{4}{\left(e^x + e^{-x}\right)^2}$.

$v\frac{du}{dx}$	**5 marks**	**Att 2**
$u\frac{dv}{dx}$	**5 marks**	**Att 2**
v^2	**5 marks**	**Att 2**
Show $\frac{dy}{dx}$	**5 marks**	**Att 2**

6 (b)

$$y = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

$$\therefore \frac{dy}{dx} = \frac{\left(e^x + e^{-x}\right)\left(e^x + e^{-x}\right) - \left(e^x - e^{-x}\right)\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}.$$

$$\therefore \frac{dy}{dx} = \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{\left(e^x + e^{-x}\right)^2} = \frac{4}{\left(e^x + e^{-x}\right)^2}.$$

OR

$v\frac{du}{dx}$	**5 marks**	**Att 2**
$u\frac{dv}{dx}$	**5 marks**	**Att 2**
v^2	**5 marks**	**Att 2**
Show $\frac{dy}{dx}$	**5 marks**	**Att 2**

6(b)

$$y=\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{e^x-\frac{1}{e^x}}{e^x+\frac{1}{e^x}}=\frac{e^{2x}-1}{e^{2x}+1}$$

$$\begin{aligned}\frac{dy}{dx}&=\frac{\left(e^{2x}+1\right)\left(2e^{2x}\right)-\left(e^{2x}-1\right)\left(2e^{2x}\right)}{\left(e^{2x}+1\right)^2}\\&=\frac{2e^{4x}+2e^{2x}-2e^{4x}+2e^{2x}}{\left(e^{2x}+1\right)^2}\\&=\frac{4e^{2x}}{\left(e^{2x}+1\right)^2}\\&=4.\frac{1}{e^{-2x}}\cdot\frac{1}{\left(e^{2x}+1\right)}\cdot\frac{1}{\left(e^{2x}+1\right)}\\&=4.\frac{1}{e^{-x}\left(e^{2x}+1\right)}\cdot\frac{1}{e^{-x}\left(e^{2x}+1\right)}\\&=4\left(\frac{1}{e^x+e^{-x}}\right)\left(\frac{1}{e^x+e^{-x}}\right)\\&=\frac{4}{\left(e^x+e^{-x}\right)^2}\end{aligned}$$

Blunders (-3)
B1 Indices
B2 Differentiation

Worthless
W1 No differentiation
W2 Integration

Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

6 (c) The function $f(x) = 2x^3 + 3x^2 + bx + c$ has a local maximum at $x = -2$.

(i) Find the value of b.

(ii) Find the range of values of c for which $f(x) = 0$ has three distinct real roots.

(i) value *b*	**5 marks**	**Att 2**
(ii) Local min at $x = 1$	**5 marks**	**Att 2**
Range *c*	**10 marks**	**Att 3**

6 (c) (i) $f(x) = 2x^3 + 3x^2 + bx + c$

$f'(x) = 6x^2 + 6x + b$

Local max at $x = -2 \quad \Rightarrow f'(-2) = 0$

$$6(-2)^2 + 6(-2) + b = 0$$
$$24 - 12 + b = 0$$
$$b = -12$$

6 (c) (ii)

$f(x) = 2x^3 + 3x^2 - 12x + c$

$f'(x) = 6x^2 + 6x - 12 = 0$ for local max/min

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2 \quad$ or $\quad x = 1$

We were given that local max is at $x = -2$, so local min is at $x = 1$

To get 3 distinct real roots, the curve must cut the x-axis 3 times.

Hence, we need the local max to be above the x-axis and the local min below it.

Local max: $x = -2$: $f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) = -16 + 12 + 24 + c = c + 20$

Above x-axis $\Rightarrow f(-2) > 0 \qquad \Rightarrow \qquad c + 20 > 0 \qquad \Rightarrow \qquad c > -20$

Local min at $x = 1$: $f(1) = 2(1)^3 + 3(1)^2 - 12(1) + c = c - 7$

Below x-axis $\Rightarrow f(1) < 0 \qquad \Rightarrow \qquad c - 7 < 0 \qquad \Rightarrow \qquad c < 7$

Thus, answer is $-20 < c < 7$

* Candidates need not explicitly state that local max and local min are on opposite sides of x-axis.

Blunders (-3)

B1 Differentiation

B2 $f'(x) \neq 0$

B3 Indices

B4 Factors

B5 Root formula once only

B6 Deducted root from factor or no deduction

B7 Inequality sign

B8 Incorrect range or no range

Slips (-1)

S1 Numerical

Worthless

W1 No Differentiation

W2 Integration

QUESTION 7

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (15, 5) marks**	**Att (5, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

7. (a)

(a) Differentiate $2x + \sin 2x$ with respect to x.

***f′*(2*x*)**	**5 marks**	**Att 2**
***f′*(sin 2*x*)**	**5 marks**	**Att 2**

7 (a)

$$f(x) = 2x + \sin 2x \;\; \Rightarrow \;\; f'(x) = 2 + 2\cos 2x.$$

Blunders (-3)
B1 Differentiation
B2 Trig formula

Attempts
A1 Error in chain rule

Worthless
W1 Integration

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

7 (b) The equation of a curve is $5x^2 + 5y^2 + 6xy = 16$.

(i) Find $\frac{dy}{dx}$ in terms of x and y.

(ii) $(1,1)$ and $(2,-2)$ are two points on the curve.
Show that the tangents at these points are perpendicular to each other.

(i) Differentiation	**5 marks**	**Att 2**
Isolate $\frac{dy}{dx}$	**5 marks**	**Att 2**
(ii) 1st slope	**5 marks**	**Att 2**
Show	**5 marks**	**Att 2**

7 (b) (i)

$$5x^2 + 5y^2 + 6xy = 16.$$

$$\therefore\ 10x + 10y\frac{dy}{dx} + 6x\frac{dy}{dx} + 6y = 0.$$

$$\therefore\ \frac{dy}{dx}(10y + 6x) = -10x - 6y \quad \Rightarrow \quad \frac{dy}{dx} = \frac{-5x - 3y}{3x + 5y}.$$

7 (b) (ii)

$$m_1 = \text{slope of tangent at } (1,1) = \frac{-5-3}{3+5} = -1.$$

$$m_2 = \text{slope of tangent at } (2,-2) = \frac{-10+6}{6-10} = 1.$$

But $m_1 . m_2 = -1$, $\therefore$ tangents are perpendicular to each other.

Blunders (-3)
B1 Differentiation
B2 Indices
B3 Incorrect value of x or no value of x
B4 Incorrect value of y or no value of y
B5 Omission of $m_1.m_2$ test

Slips (-1)
S1 Numerical

Attempts
A1 Error in differentiation formula
A2 $\frac{dy}{dx} = 10x + 10y\frac{dy}{dx} + 6x\frac{dy}{dx} + 6y$
And uses the three $\left(\frac{dy}{dx}\right)$ terms

Worthless
W1 No differentiation
W2 Integration

Part (c) **20 (15, 5) marks** **Att (5, 2)**

7 (c) Let $y = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$.

Find $\dfrac{dy}{dx}$ and express it in the form $\dfrac{a}{a+x^b}$, where $a, b \in \mathbf{N}$.

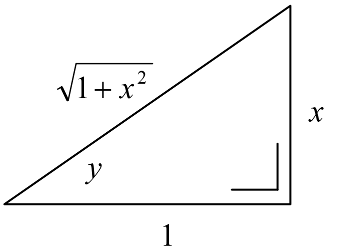
Tan *y* = *x* **5 marks** **Att 2**
Differentiate **15 marks** **Att 5**

7(c) $y = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right) \Rightarrow \sin y = \dfrac{x}{\sqrt{1+x^2}}$

$$\tan y = \frac{x}{1} = x$$

$$y = \tan^{-1} x$$

$$\therefore \frac{dy}{dx} = \frac{1}{1+x^2}$$



OR

Differentiation **15 marks** **Att 5**
Other work **5 marks** **Att 2**

7 (c)

$$y = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$$

$$\sin y = \frac{x}{\sqrt{1+x^2}} = \frac{x}{\left(1+x^2\right)^{\frac{1}{2}}}$$

$$\cos y.\frac{dy}{dx} = \frac{\left(1+x^2\right)^{\frac{1}{2}}(1) - x\left[\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}} \cdot 2x\right]}{\left(1+x^2\right)}$$

$$= \frac{\left(1+x^2\right)^{\frac{1}{2}} - \dfrac{x^2}{\left(1+x^2\right)^{\frac{1}{2}}}}{\left(1+x^2\right)^{\frac{3}{2}}}$$

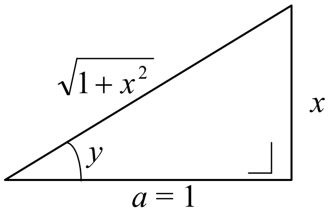
$$= \frac{1+x^2-x^2}{\left(1+x^2\right)^{\frac{3}{2}}}$$

$$\cos y.\frac{dy}{dx} = \frac{1}{\left(1+x^2\right)^{\frac{3}{2}}}$$

$$\frac{dy}{dx} = \frac{1}{\cos y} \cdot \frac{1}{\left(1+x^2\right)^{\frac{3}{2}}}$$

$$= \left(1+x^2\right)^{\frac{1}{2}}.\frac{1}{\left(1+x^2\right)^{\frac{3}{2}}}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$



$$a^2 + x^2 = \left(\sqrt{1+x^2}\right)^2$$

$$a^2 + x^2 = 1 + x^2$$

$$a^2 = 1$$

$$a = 1$$

$$\sin y = \frac{x}{\left(1+x^2\right)^{\frac{1}{2}}}$$

$$\cos y = \frac{1}{\left(1+x^2\right)^{\frac{1}{2}}}$$

$$\frac{1}{\cos y} = \frac{\left(1+x^2\right)^{\frac{1}{2}}}{1} = \left(1+x^2\right)^{\frac{1}{2}}$$

OR

Differentiation	**15 marks**	**Att 5**
Correct form	**5 marks**	**Att 2**

7 (c)

$$y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right).$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\frac{x^2}{1+x^2}}} \times \frac{1\sqrt{1+x^2} - x.\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}.2x}{1+x^2}.$$

$$\therefore \frac{dy}{dx} = \frac{\sqrt{1+x^2}}{1} \times \frac{1+x^2-x^2}{\left(1+x^2\right)^{\frac{3}{2}}} = \frac{1}{1+x^2}.$$

Blunders (-3)

B1 Incorrect sin y

B2 Differentiation

B3 Error value of cos y

B4 Definition of sin y and/or cos y (once only)

B5 Sides of triangle (once only)

B6 Indices

Attempts

A1 Error in differentiation formula

Worthless

W1 Integration

QUESTION 8

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

8. (a)

(a) Find $\int (2x+\cos 3x)dx$

$\int 2xdx$	**5 marks**	**Att 2**
$\int \cos 3x.dx$	**5 marks**	**Att 2**

8 (a)

$$\int (2x+\cos 3x)dx = x^2+\frac{1}{3}\sin 3x+\text{constant.}$$

Blunders (-3)

B1 Integration

B2 Indices

B3 No *c* penalise 2nd element.

Attempts

A1 Only *c* correct (on 2nd element only)

Worthless

W1 Differentiation for integration

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

8 (b)

Evaluate **(i)** $\int_0^1 3x^2 e^{x^3}\,dx$ **(ii)** $\int_2^4 \frac{2x^3}{x^2-1}dx$.

(i) Integration	**5 marks**	**Att 2**
Value	**5 marks**	**Att 2**
(ii) Integration	**5 marks**	**Att 2**
Value	**5 marks**	**Att 2**

8 (b) (i)

$$\int_0^1 3x^2 e^{x^3}\,dx \qquad \text{Let } u = e^{x^3}.\ \therefore\ du = 3x^2 e^{x^3}\,dx.$$

$$\therefore\ \int_0^1 3x^2 e^{x^3}\,dx = \int_1^e du = [u]_1^e = e-1.$$

OR

8 (b) (i)

$$\int_0^1 3x^2.e^{x^3}\,dx \qquad \text{Let } u = x^3$$

$$= \int e^{x^3}\left(3x^2dx\right) \qquad \frac{du}{dx} = 3x^2$$

$$= \int e^u.du \qquad du = 3x^2.dx$$

$$= e^u$$

$$= e^{x^3}\Big]_0^1 = e^1 - e^0 = (e-1)$$

8 (b) (ii)

$$\int_2^4 \frac{2x^3}{x^2-1}dx \qquad \text{Let } u = x^2-1.\ \therefore du = 2xdx.$$

$$\int_2^4 \frac{2x^3}{x^2-1}dx = \int_3^{15}\frac{u+1}{u}du = \int_3^{15}\left(1+\frac{1}{u}\right)du = \left[u+\log_e u\right]_3^{15}$$

$$= 15-3+\log_e 15-\log_e 3 = 12+\log_e 5.$$

OR

8 (b) (ii)

$$2\int_{2}^{4}\frac{x^3}{x^2-1}.dx$$

$$=2\int\left[x+\frac{x}{x^2-1}\right]dx$$

$$=2\left[\int xdx+\int\frac{xdx}{x^2-1}\right]$$

$$=2\left[\frac{x^2}{2}+\int\frac{du}{2u}\right]$$

$$=2\left[\frac{x^2}{2}+\frac{1}{2}\int\frac{du}{u}\right]$$

$$=2\left[\frac{x^2}{2}+\frac{1}{2}\ln u\right]$$

$$=x^2+\ln\left(x^2-1\right)\Big]_2^4$$

$$=\left(16+\ln 15\right)-\left(4+\ln 3\right)$$

$$=12+\ln\left(\frac{15}{3}\right)$$

$$=12+\ln 5$$

$$\begin{array}{r} x \\ x^2-1\overline{\smash{)}\,x^3} \\ \underline{x^3-x} \\ x \end{array}$$

Let $u=x^2-1$

$$\frac{du}{dx}=2x$$

$$du=2x.dx$$

$$\frac{du}{2}=xdx$$

* Incorrect substitution and unable to finish yields attempt at most.

Blunders (-3)
B1 Integration
B2 Indices
B3 Differentiation
B4 Limits
B5 Incorrect order in applying limits
B6 Not calculating substituted limits
B7 Not changing limits
B8 Error logs

Slips (-1)
S1 Numerical
S2 Trig value
S3 $e^0\neq 1$
S4 Answer not tidied up

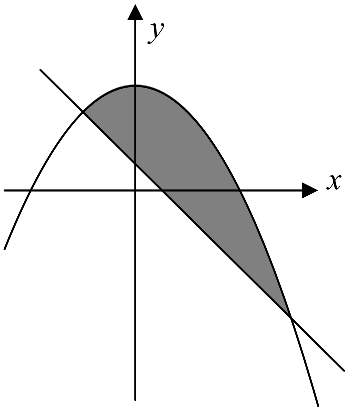
Worthless
W1 Differentiation instead of integration except where other work merits attempt.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

8 (c)

(c) The diagram shows the curve $y = 4 - x^2$ and the line $2x + y - 1 = 0$.

Calculate the area of the shaded region enclosed by the curve and the line.



Points of intersection	**5 marks**	**Att 2**
First integrand	**5 marks**	**Att 2**
Second integrand	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

Points of intersection:

$$4 - x^2 = 1 - 2x$$

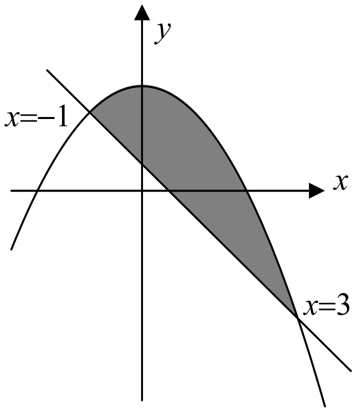
$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = -1, \quad x = 3$$

$$\therefore \text{Area} = \int_{-1}^{3} \left[(4 - x^2) - (1 - 2x)\right] dx = \int_{-1}^{3} \left(3 + 2x - x^2\right) dx$$

$$= \left[3x + x^2 - \frac{x^3}{3}\right]_{-1}^{3} = \left(9 + 9 - 9\right) - \left(-3 + 1 + \tfrac{1}{3}\right) = 10\tfrac{2}{3}$$



OR

Relevant Points	**5 marks**	**Att 2**
Area above *x*-axis	**5 marks**	**Att 2**
Area below *x*-axis	**5 marks**	**Att 2**
Total Area	**5 marks**	**Att 2**

8 (c)

$$2x + y - 1 = 0 \Rightarrow y = 1 - 2x.$$

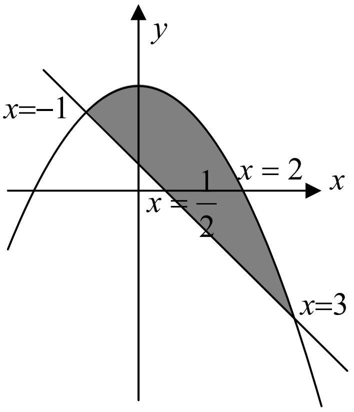
$$y = 4 - x^2 \Rightarrow 1 - 2x = 4 - x^2$$

$$\therefore\ x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0.$$

$$\therefore x = 3 \text{ or } x = -1.$$

$$2x + y - 1 = 0 \text{ cuts } x\text{-axis at } x = \frac{1}{2}.$$

$$y = 4 - x^2 \text{ cuts } x\text{-axis at } x = \pm 2.$$



$$\text{Shaded region above } x\text{-axis } = \int_{-1}^{2}\left(4 - x^2\right)dx - \int_{-1}^{\frac{1}{2}}(1-2x)dx.$$

$$= \left[4x - \frac{1}{3}x^3\right]_{-1}^{2} - \left[x - x^2\right]_{-1}^{\frac{1}{2}}$$

$$= \left[\left(8 - \frac{8}{3}\right) - \left(-4 + \frac{1}{3}\right)\right] - \left[\left(\frac{1}{2} - \frac{1}{4}\right) - (-1-1)\right].$$

$$= \left|9 - 2\frac{1}{4}\right| = 6\frac{3}{4}.$$

$$\text{Shaded region below } x\text{-axis } = \int_{\frac{1}{2}}^{3}(1-2x)dx - \int_{2}^{3}\left(4 - x^2\right)dx.$$

$$= \left[x - x^2\right]_{\frac{1}{2}}^{3} - \left[4x - \frac{1}{3}x^3\right]_{2}^{3}$$

$$= \left[(3-9) - \left(\frac{1}{2} - \frac{1}{4}\right)\right] - \left[(12-9) - \left(8 - \frac{8}{3}\right)\right]$$

$$= \left[-6 - \frac{1}{4}\right] - \left[3 - 5\frac{1}{3}\right]$$

$$= \left|-6\frac{1}{4} + 2\frac{1}{3}\right| = \frac{47}{12}.$$

$$\text{Total shaded region } = \frac{27}{4} + \frac{47}{12} = \frac{128}{12} = \frac{32}{3}.$$

Blunders (-3)

B1 Integration
B2 Indices
B3 Factors once only
B4 Calculation of point of intersection of line and curve
B5 Calculation of points where line cuts x-axis
B6 Calculation of points where curve cuts x-axis
B7 Error in area triangle
B8 Error in area formula
B9 Incorrect order in applying limits
B10 Not calculating substituted limits
B11 Error with line
B12 Error with curve
B13 Uses $\pi\int ydx$ for area formula

Attempts

A1 Uses volume formula
A2 Uses y^2 in formula

Worthless

W1 Differentiation instead of integration except where other work merits attempt
W2 Wrong area formula and no work.

Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2008

MARKING SCHEME

MATHEMATICS – PAPER 2

HIGHER LEVEL

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2008

MATHEMATICS – HIGHER LEVEL – PAPER 2

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:

• Blunders	- mathematical errors/omissions	(-3)
• Slips	- numerical errors	(-1)
• Misreadings	(provided task is not oversimplified)	(-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work of merit is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* part of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**
Part (c)	**25 (5, 5, 5, 5, 5) marks**	**Att (2, 2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

1. (a) A circle with centre (–3, 2) passes through the point (1, 3).
Find the equation of the circle.

(a) Radius/Centre **5 marks** **Att 2**
Finish **5 marks** **Att 2**

1. (a) Centre of circle is $c(-3, 2)$ and p is $(1, 3)$.

$$|ap| = r = \sqrt{(-3-1)^2 + (2-3)^2} = \sqrt{16+1} = \sqrt{17}.$$

$$\therefore \text{ Equation of circle: } (x+3)^2 + (y-2)^2 = 17.$$

Blunders (-3)
B1 Error in distance formula
B2 Error in circle formula

Slips (-1)
S1 Uses (-3, 2) as point on circle and uses (1, 3) as centre

Attempts
A1 Writes down correct equation of a circle and stops

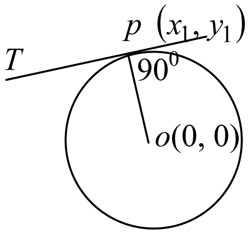
Part (b) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

(b) **(i)** Prove that the equation of the tangent to the circle $x^2 + y^2 = r^2$ at the point (x_1, y_1) is $xx_1 + yy_1 = r^2$.

(ii) A tangent is drawn to the circle $x^2 + y^2 = 13$ at the point $(2, 3)$.
This tangent crosses the *x*-axis at $(k, 0)$. Find the value of *k*.

(b)(i) Equation *T* **5 marks** **Att 2**
Finish **5 marks** **Att 2**

1. (b) (i)



Slope $op = \frac{y_1}{x_1} \Rightarrow$ slope $T = -\frac{x_1}{y_1}$.

$\therefore$ Equation of tangent T: $y - y_1 = -\frac{x_1}{y_1}(x - x_1)$.

$\therefore\ yy_1 - y_1^2 = -xx_1 + x_1^2 \Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2$.

But $(x_1, y_1) \in x^2 + y^2 = r^2 \Rightarrow x_1^2 + y_1^2 = r^2$.

$\therefore\ xx_1 + yy_1 = r^2$.

Blunders (-3)

B1 Error in finding slope of T.
B2 Error in finding equation of tangent
B3 Error in showing $(x_1, y_1) \in x^2 + y^2 = r^2 \Rightarrow x_1^2 + y_1^2 = r^2$.

(b) (ii) **5 marks** **Att 2**

1. (b) (ii) Tangent at (2, 3) is $2x + 3y = 13$

$y = 0 \Rightarrow x = 6\frac{1}{2}$. $\therefore\ k = 6\frac{1}{2}$.

Blunders (-3)

B1 Error in applying formula
B2 Transposition error
B3 Wrong axis

Slips (-1)

S1 Calculation errors

Attempts

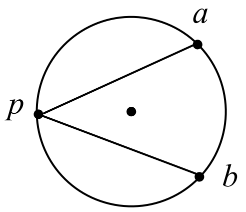
A1 Correct linear formula written down with some correct substitution and stops

Part (c) **25 (5, 5, 5, 5, 5) marks** **Att (2, 2, 2, 2, 2)**

1. (c) A circle passes through the points $a(8, 5)$ and $b(9, -2)$.
The centre of the circle lies on the line $2x - 3y - 7 = 0$.

(i) Find the equation of the circle.

(ii) p is a point on the major arc ab of the circle.
Show that $|\angle apb| = 45^\circ$.



(c)(i) Two equations **5 marks** **Att 2**
Solve **5 marks** **Att 2**
Finish **5 marks** **Att 2**

(c) (i) Let circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.

$a(8,5) \in \text{circle} \Rightarrow 64 + 25 + 16g + 10f + c = 0. \ \therefore\ 16g + 10f + c = -89.$

$b(9,-2) \in \text{circle} \Rightarrow 81 + 4 + 18g - 4f + c = 0. \ \therefore 18g - 4f + c = -85.$

Centre $(-g, -f) \in 2x - 3y - 7 = 0 \Rightarrow -2g + 3f = 7.$

$$\begin{array}{l} 16g + 10f + c = -89 \\ \underline{18g - 4f + c \ = -85} \\ -2g + 14f = -4 \end{array}$$

$$\text{But } \underline{-2g + 3f = 7}$$

$$11f = -11 \Rightarrow f = -1. \ -2g + 3f = 7 \Rightarrow -2g = 10 \Rightarrow g = -5.$$

$16g + 10f = c = -89 \Rightarrow -80 - 10 + c = -89 \Rightarrow c = 1.$

$\therefore$ Equation of circle : $x^2 + y^2 - 10x - 2y + 1 = 0.$

Blunders (-3)
B1 Error in finding equation.
B2 Error in formula for the equation of the circle

Slips (-1)
S1 Calculation errors

(c)(ii) Two slopes **5 marks** **Att 2**
Finish **5 marks** **Att 2**

1. (c) (ii) Label the centre c.

$c(5,1),\ a(8,5),\ b(9,-2).$

Slope $ac = \frac{5-1}{8-5} = \frac{4}{3}$, slope $bc = \frac{-2-1}{9-5} = \frac{-3}{4}$.

$\frac{4}{3} \times \frac{-3}{4} = -1 \ \Rightarrow |\angle acb| = 90^\circ.$

But $|\angle acb| = 2|\angle apb| \ \Rightarrow |\angle apb| = 45^\circ.$

Blunders (-3)
B1 Error in finding measure of angle at centre
B2 Error in finding required angle

Slips (-1)
S1 Error in calculations

QUESTION 2

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

2. (a)

(a) Given that $\left|10\vec{i}+k\vec{j}\right| = \left|11\vec{i}-2\vec{j}\right|$, find the two possible values of $k \in \mathbf{R}$.

(a) Mods	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

2. (a)

$$\left|10\vec{i}+k\vec{j}\right| = \left|11\vec{i}-2\vec{j}\right|. \quad \therefore \sqrt{100+k^2} = \sqrt{121+4} \;\Rightarrow\; k^2 = 25 \;\Rightarrow\; k = \pm 5.$$

Blunders (-3)

B1 Error in expression for mod of vector

B2 Error in solving equation

B3 One value not given.

Slips (-1)

S1 Error in calculations

Attempts

A1 Gives correct expression for mod of vector and stops

Part (b) **20 (10, 10) marks** **Att (3, 3)**

2 **(b)** $\vec{x} = -\vec{i} + 3\vec{j},\ \vec{y} = 4\vec{i} - 2\vec{j}$ and $\vec{z} = \vec{x} - t\vec{y}$, where $t \in \mathbf{R}$.

(i) Given that $\vec{x} \perp \vec{z}$, calculate the value of t.

(ii) Find the measure of $\angle xoy$, where o is the origin.

(b)(i) **10 marks** **Att 3**

$$\vec{z} = \vec{x} - t\vec{y} = -\vec{i} + 3\vec{j} - 4t\vec{i} + 2t\vec{j}$$

$$\therefore \vec{z} = (-1-4t)\vec{i} + (3+2t)\vec{j}.$$

But $\vec{x} \perp \vec{y} \ \Rightarrow\ \vec{x}.\vec{y} = 0.$

$$\therefore \left(-\vec{i} + 3\vec{j}\right)\left[(-1-4t)\vec{i} + (3+2t)\vec{j}\right] = 0.$$

$$\therefore 1 + 4t + 9 + 6t = 0 \ \Rightarrow\ t = -1.$$

Blunders (-3)

B1 Error in expressing in terms of $\vec{i}$ *and* $\vec{j}$.

B2 Error in Scalar Product property

B3 Error in solving equation

Slips (-1)

S1 Error in calculations

(b) (ii) **10 marks** **Att 3**

2. (b) (ii)

$$\cos\angle xoy = \frac{\overrightarrow{ox}\,.\,\overrightarrow{oy}}{\left|\overrightarrow{ox}\right|.\left|\overrightarrow{oy}\right|} = \frac{\left(-\vec{i} + 3\vec{j}\right)\left(4\vec{i} - 2\vec{j}\right)}{\left|-\vec{i} + 3\vec{j}\right|\left|4\vec{i} - 2\vec{j}\right|}.$$

$$\cos\angle xoy = \frac{-4-6}{\sqrt{10}\sqrt{20}} = \frac{-10}{10\sqrt{2}} = \frac{-1}{\sqrt{2}}.$$

$$\therefore\ \left|\angle xoy\right| = 135^\circ.$$

Blunders (-3)

B1 Error in setting up the equation

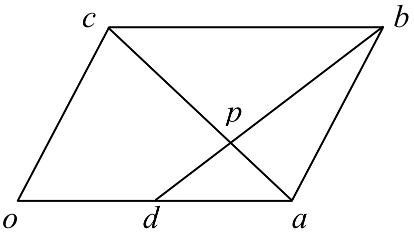
B2 Error in solving equation

Slips (-1)

S1 Error in calculations

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

2. (c) *oabc* is a parallelogram, where *o* is the origin. *d* is the midpoint of [*oa*] and [*db*] cuts the diagonal [*ac*] at *p*.



(i) Given that $\overrightarrow{ap} = k\,\overrightarrow{ac}$, where $k \in \mathbf{R}$, express $\vec{p}$ in terms of $\vec{a}$, $\vec{c}$ and *k*.

(ii) Given that $\overrightarrow{bp} = l\,\overrightarrow{bd}$, where $l \in \mathbf{R}$, express $\vec{p}$ in terms of $\vec{a}$, $\vec{c}$ and *l*.

(iii) Hence find the value of *k* and the value of *l*.

(c) (i) **10 marks** **Att 3**

2 (c) (i)

$$\overrightarrow{ap} = k\,\overrightarrow{ac} \Rightarrow \vec{p} - \vec{a} = k\left(\vec{c} - \vec{a}\right) \Rightarrow \vec{p} = k\,\vec{c} - k\,\vec{a} + \vec{a}.$$

$$\therefore\ \vec{p} = (1-k)\vec{a} + k\,\vec{c}.$$

Blunders (-3)

B1 Error in simplifying $\overrightarrow{ap} = k\,\overrightarrow{ac}$,

B2 Error in transposing

(c) (ii) **5 marks** **Att 2**

2. (c) (ii)

$$\overrightarrow{bp} = l\,\overrightarrow{bd} \Rightarrow \vec{p} - \vec{b} = l\left(\vec{d} - \vec{b}\right) \Rightarrow \vec{p} = l\,\vec{d} + (1-l)\vec{b} \Rightarrow \vec{p} = \frac{1}{2}l\,\vec{a} + (1-l)\left(\vec{a} + \vec{c}\right).$$

$$\therefore\ \vec{p} = \left(1 - \frac{1}{2}l\right)\vec{a} + (1-l)\vec{c}.$$

Blunders (-3)

B1 Error in simplifying $\overrightarrow{bp} = l\,\overrightarrow{bd}$,

B2 Error in finishing.

(c) (iii) **5 marks** **Att 2**

2. (c) (iii)

$$\vec{p} = (1-k)\vec{a} + k\,\vec{c} \text{ and } \vec{p} = \left(1 - \frac{1}{2}l\right)\vec{a} + (1-l)\vec{c}.$$

$$\therefore\ 1-k = 1 - \frac{1}{2}l \Rightarrow l = 2k \text{ and } k = 1-l.$$

$$\therefore\ k = 1-2k \Rightarrow k = \frac{1}{3} \text{ and } l = \frac{2}{3}.$$

Blunders (-3)
B1 Error in setting up equations
B2 Error in solving equations

Slips (-1)
S1 Errors in calculations

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (- , 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

3. (a) The parametric equations $x = 7t - 4$ and $y = 3 - 3t$ represent a line, where $t \in \mathbf{R}$. Find the Cartesian equation of the line.

(a) **10 marks** **Att 3**

3. (a)

$$\begin{aligned} x = 7t - 4 \;&\Rightarrow\; 3x = 21t - 12 \\ y = 3 - 3t \;&\Rightarrow\; 7y = 21 - 21t \\ \therefore\; & 3x + 7y = 9. \end{aligned}$$

Blunders (-3)

B1 Error in setting up equations

B2 Error in solving the equations

Slips (-1)

S1 Errors in calculations

Part (b) **20 (5, 5, 5, 5) marks** **Att (- , 2, 2, 2)**

3. (b) $a(2,1)$, $b(10,7)$, $c(14,10)$ and $d(7,1)$ are four points.

(i) Plot *a*, *b*, *c* and *d* on the co-ordinate plane.

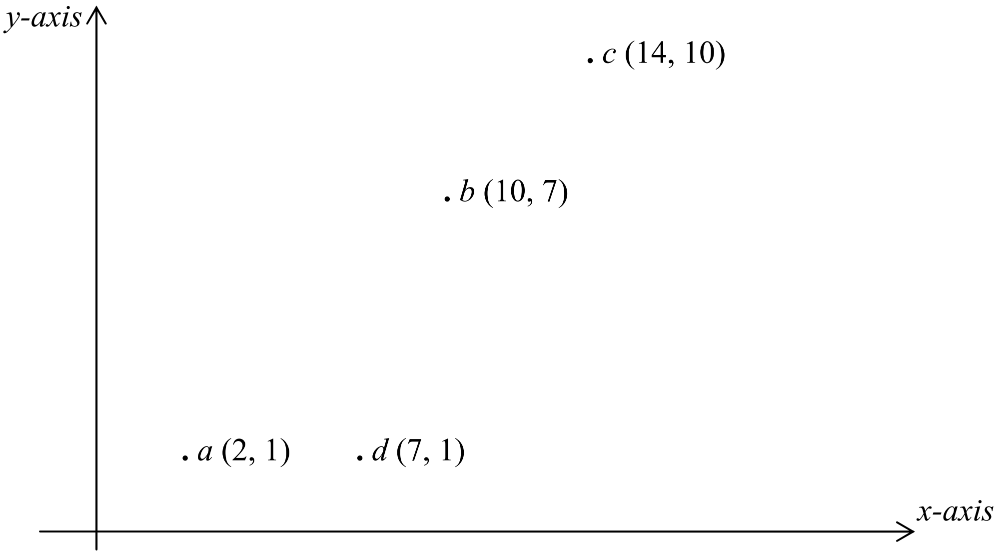
(ii) Verify that $|ab| = 2|bc|$ and $|ab| = 2|ad|$.

(iii) Find a', b', c' and d', the respective images of *a*, *b*, *c* and *d* under the transformation $f:(x,y) \rightarrow (x',y')$, where $x' = x + y$ and $y' = x - 2y$.

(iv) Verify that $|a'b'| = 2|b'c'|$ but that $|a'b'| \neq 2|a'd'|$.

(b) (i) **5 marks** **Hit / Miss**

3.(b) (i)



* All four points correct: 5 marks. Otherwise, 0 marks.

(b) (ii) **5 marks** **Att 2**

(b) (ii)

$a(2,1),\ b(10,7),\ \ c(14,10),\ \ d(7,1).$

$|ab| = \sqrt{(10-2)^2 + (7-1)^2} = \sqrt{64+36} = 10.$

$|bc| = \sqrt{(10-14)^2 + (7-10)^2} = \sqrt{16+9} = 5.$

$\therefore\ \ |ab| = 2|bc|.$

$|ad| = \sqrt{(2-7)^2 + (1-1)^2} = \sqrt{25+0} = 5.$

$\therefore\ |ab| = 2|ad|.$

Blunders (-3)

B1 Error in distance formula

B2 Incorrect squaring

Slips (-1)

S1 Error in calculations

(b) (iii) **5 marks** **Att 2**

3. (b) (iii)

$a(2,1),\ b(10,7),\ \ c(14,10),\ \ d(7,1).\quad x' = x + y$ and $y' = x - 2y.$

$a' = f(2,1) \quad \Rightarrow a' = (3,0).\qquad b' = f(10,7) \ \Rightarrow b' = (17,-4).$

$c' = f(14,10) \Rightarrow c' = (24,-6).\qquad d' = f(7,1) \quad \Rightarrow d' = (8,5).$

Blunders (-3)

B1 Any error in finding images

Part (b) (iv) **5 marks** **Att 2**

3. (b) (iv)

$|a'b'| = \sqrt{(3-17)^2 + (0+4)^2} = \sqrt{196+16} = \sqrt{212} = 2\sqrt{53}.$

$|b'c'| = \sqrt{(17-24)^2 + (-4+6)^2} = \sqrt{49+4} = \sqrt{53}.$

$|a'd'| = \sqrt{(3-8)^2 + (0-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}.$

$\therefore\ \ |a'b'| = 2|b'c'|$ but $|a'b'| \neq 2|a'd'|.$

Blunders (-3)

B1 Error in distance formula

B2 Incorrect squaring

Slips (-1)

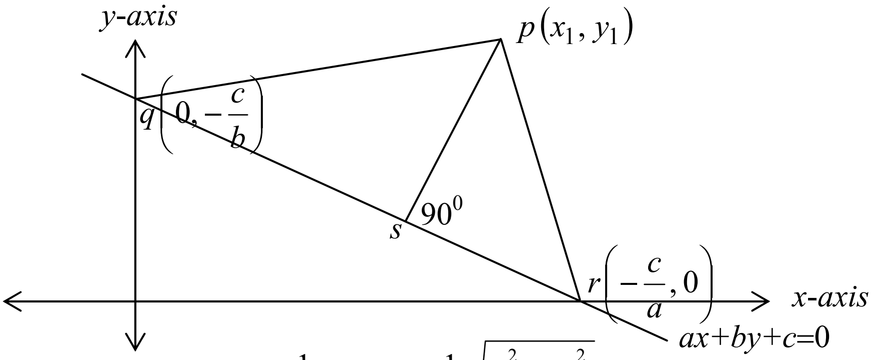
S1 Error in calculations

Part (c) 20 (5, 5, 5, 5) marks Att (2, 2, 2, 2)

3.(c) Prove that the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is $\dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

(c) Diagram	**5 marks**	**Att 2**
Area of *pqr*	**5 marks**	**Att 2**
Area of image	**5 marks**	**Att 2**
Solves equation	**5 marks**	**Att 2**

3. (c)



$$\text{Area triangle } pqr = \frac{1}{2}|qr|.|ps| = \frac{1}{2}\sqrt{\frac{c^2}{a^2} + \frac{c^2}{b^2}}.|ps|$$

$$= \frac{1}{2}\left|\frac{c}{ab}\right|.\sqrt{a^2 + b^2}.|ps|.$$

Translating $q\left(0, -\frac{c}{b}\right)$ to $(0,0) \Rightarrow p(x_1, y_1) \to \left(x_1, y_1 + \frac{c}{b}\right)$ and $r\left(-\frac{c}{a}, 0\right) \to \left(-\frac{c}{b}, \frac{c}{b}\right)$.

$$\therefore \text{ Area triangle } pqr = \frac{1}{2}\left|x_1\left(\frac{c}{b}\right) - \left(-\frac{c}{a}\right)\left(y_1 + \frac{c}{b}\right)\right| = \frac{1}{2}\left|\frac{cx_1}{b} + \frac{cy_1}{a} + \frac{c^2}{ab}\right|.$$

$$= \frac{1}{2}\left|\frac{acx_1 + bcy_1 + c^2}{ab}\right| = \frac{1}{2}\left|\frac{c}{ab}\right||ax_1 + by_1 + c|.$$

$$\therefore \sqrt{a^2 + b^2}.|ps| = |ax_1 + by_1 + c| \Rightarrow |ps| = \perp \text{distance} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Blunders (-3)
B1 Error in diagram
B2 Error in area each time
B3 Error in setting up or solving equation

Slips (-1)
S1 Error in calculations

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, - , - , -)**

Part (a)	**10 marks**	**Att 3**

4. (a)

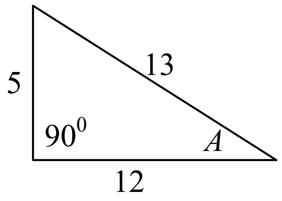
(a) A and B are acute angles such that $\tan A = \frac{5}{12}$ and $\tan B = \frac{3}{4}$.

Find $\cos(A - B)$, as a fraction.

(a)	**10 marks**	**Att 3**

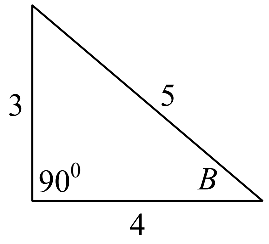
4. (a)

$\tan A = \frac{5}{12}$



$\therefore \sin A = \frac{5}{13}, \quad \cos A = \frac{12}{13}.$

$\tan B = \frac{3}{4}$



$\therefore \sin B = \frac{3}{5}, \quad \cos B = \frac{4}{5}.$

$$\therefore \cos(A - B) = \cos \text{A} \cos B + \sin A \sin B = \frac{12}{13}.\frac{4}{5} + \frac{5}{13}.\frac{3}{5} = \frac{63}{65}.$$

Blunders (-3)

B1 Error in finding sinA or cosAor sinB or cosB each time

B2 Sign error in cos(A – B)

Slips (-1)

S1 Error in calculations

Attempts

A1 Draws a right angled triangle with length of one side indicated

A2 Evaluates A and B and subtracts

Part (b) **20 (10, 10) marks** **Att (3, 3)**

4. **(b)** **(i)** Show that $\frac{\sin 2A}{1+\cos 2A} = \tan A$.

(ii) Hence, or otherwise, prove that $\tan 22\tfrac{1}{2}^{\circ} = \sqrt{2}-1$.

(b) (i) **10 marks** **Att 3**

4. (b) (i)

$$\frac{\sin 2A}{1+\cos 2A} = \frac{2\text{sinAcos}A}{2\cos^2 A} = \frac{\sin A}{\cos A} = \tan A.$$

Blunders (-3)

B1 Error in simplifying sin2A or 1+cos2A

B2 Error in finding tanA.

(b) (ii) **10 marks** **Att 3**

4. (b) (ii)

$$\tan 22\tfrac{1}{2}^{\circ} = \frac{\sin 45^{\circ}}{1+\cos 45^{\circ}} = \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}+1}$$

$$= \frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1.$$

Blunders (-3)

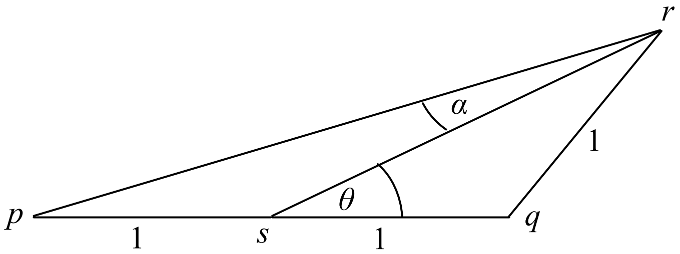
B1 Fails to link double and half angle correctly

B2 Error in evaluation

B3 Answer not in required form

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, - , - , -)**

(c) In the triangle pqr, $|\angle rsq| = \theta^\circ$, $|\angle prs| = \alpha^\circ$, $|rq| = 1$, $|ps| = 1$ and $|sq| = 1$.



(i) Find $|sr|$ in terms of θ.

(ii) Hence, or otherwise, show that $\tan\theta = 3\tan\alpha$.

(c) (i)	**5 marks**	**Att 2**

4 (c) (i)

In triangle qrs, $\angle srq = \theta$ as $|sq| = |qr|$. $\therefore \angle sqr = 180^\circ - 2\theta$.

$$\frac{|sr|}{\sin(180^\circ - 2\theta)} = \frac{1}{\sin\theta}$$

$$\Rightarrow |sr| = \frac{\sin 2\theta}{\sin\theta} = \frac{2\sin\theta\cos\theta}{\sin\theta} = 2\cos\theta.$$

Blunders (-3)

B1 Error in applying sine rule/cosine rule each time.

B2 Error in simplifying $\sin(180^\circ - 2\theta)$ or $\cos(180^\circ - 2\theta)$

B3 Error in solving equation each time

Slips (-1)

S1 Error in calculations

(c)(ii) Set up Sine Rule	**5 marks**	**Hit/Miss**
Expand sin(θ–α)	**5 marks**	**Hit/Miss**
Finish	**5 marks**	**Hit/Miss**

4. (c) (ii) In the triangle psr, $\angle rps = \theta - \alpha$.

$$\therefore \frac{\sin(\theta - \alpha)}{2\cos\theta} = \frac{\sin\alpha}{1} \Rightarrow \sin(\theta - \alpha) = 2\cos\theta\sin\alpha.$$

$$\therefore \sin\theta\cos\alpha - \cos\theta\sin\alpha = 2\cos\theta\sin\alpha \Rightarrow \sin\theta\cos\alpha = 3\cos\theta\sin\alpha$$

Dividing across by $\cos\theta\cos\alpha$ results in

$$\frac{\sin\theta}{\cos\theta} = \frac{3\sin\alpha}{\cos\alpha}.$$

$$\therefore \tan\theta = 3\tan\alpha.$$

* Second 5 marks only available if first 5 has been awarded.
Third 5 marks only available if second 5 has been awarded.

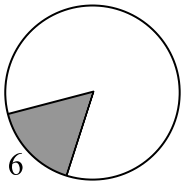
QUESTION 5

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 15) marks**	**Att (- , 5)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

5. (a)

In the shaded sector in the diagram, the arc is 6 cm long, and the angle of the sector 0·75 radians.
Find the area of the sector.



(a) Radius	**5 marks**	**Att 2**
Area	**5 marks**	**Att 2**

5. (a) Length of arc $= r\theta \;\Rightarrow\; r(0.75) = 6 \;\Rightarrow\; r = 8\text{cm}.$

Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 64 \times (0.75) = 24\text{cm}^2.$

Blunders (-3)

B1 Error in calculating radius

B2 Error in calculating area

Slips (-1)

S1 No units

Attempts

A1 Correct formula and some correct substitution and stops

Part (b) **20 (5, 15) marks** **Att (- , 5)**

(b) **(i)** Express $\sin 4x - \sin 2x$ as a product.

(ii) Find all the solutions of the equation $\sin 4x - \sin 2x = 0$ in the domain $0^\circ \le x \le 180^\circ$.

(b) (i) **5 marks** **Hit / Miss**

5. (b) (i)

$$\sin 4x - \sin 2x = 2\cos 3x \sin x.$$

(b) (ii) **15 marks** **Att 5**

5. (b) (ii)

$$\sin 4x - \sin 2x = 0 \;\Rightarrow\; 2\cos 3x \sin x = 0.$$

$$\therefore\; \cos 3x = 0 \text{ or } \sin x = 0.$$

$$\therefore\; 3x = 90^\circ,\ 270^\circ,\ 450^\circ \quad \text{or} \quad x = 0^\circ,\ 180^\circ.$$

$$\therefore\; x = 30^\circ,\ 90^\circ, 150^\circ \quad \text{or} \quad x = 0^\circ, 180^\circ.$$

Solution is $\{0^\circ, 30^\circ, 90^\circ, 150^\circ, 180^\circ\}$.

Blunders (-3)

B1 Error in solving equation

B2 Solutions omitted

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

(c) A triangle has sides of lengths *a*, *b* and *c*. The angle opposite the side of length *a* is *A*.

(i) Prove that $a^2 = b^2 + c^2 - 2bc\cos A$.

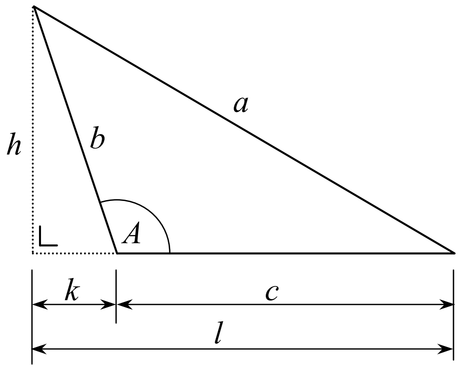
(ii) If *a*, *b* and *c* are consecutive whole numbers, show that

$$\cos A = \frac{a+5}{2a+4}.$$

(c)(i) Diagrams	**5 marks**	**Att 2**
Value for *l*	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

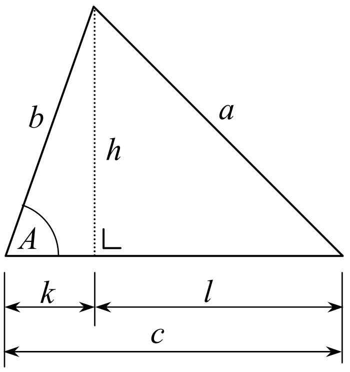
5 (c) (i)

case 1: A obtuse



$$k = b\cos(180° - A) = -b\cos A$$
$$\therefore l = c + k = c - b\cos A$$
$$h = b\sin(180° - A) = b\sin A$$

case 2: A acute



$$k = b\cos A$$
$$\therefore l = c - k = c - b\cos A$$
$$h = b\sin A$$

Both cases continue:

by Pythagoras' theorem: $a^2 = h^2 + l^2$

$$a^2 = (b\sin A)^2 + (c - b\cos A)^2$$
$$a^2 = b^2\sin^2 A + c^2 + b^2\cos^2 A - 2bc\cos A.$$
$$a^2 = b^2\left(\sin^2 A + \cos^2 A\right) + c^2 - 2bc\cos A$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$

* Correct acute case but omits or mishandles obtuse case, or vice versa: one blunder.

Blunders (-3)

B1 Error in diagram(s)

B2 Error in finding *l*

B3 Error in finishing

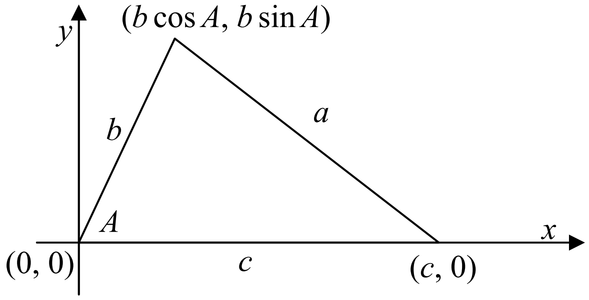
Attempts

A1 Draws diagram with correct labelling.

Or

(c)(i) Diagram	**5 marks**	**Att 2**
Substitutes in formula	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

(covering all cases)



Distance formula $\Rightarrow$ $a = \sqrt{(c - b\cos A)^2 + (0 - b\sin A)^2}$

$$a^2 = c^2 - 2bc\cos A + b^2\cos^2 A + b^2\sin^2 A$$
$$= c^2 - 2bc\cos A + b^2(\cos^2 A + \sin^2 A)$$
$$= b^2 + c^2 - 2bc\cos A.$$

Blunders (-3)

B1 Error in diagram

B2 Error in use of distance formula

B3 Error in finishing

Attempts

A1 Draws diagram with correct labelling.

(c) (ii)	**5 marks**	**Att 2**

5. (c) (ii)

a, b and c are consecutive whole numbers,

$\therefore\ b = a + 1$ and $c = a + 2$.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad \Rightarrow \quad \cos A = \frac{(a+1)^2 + (a+2)^2 - a^2}{2(a+1)(a+2)}.$$

$$\cos A = \frac{a^2 + 2a + 1 + a^2 + 4a + 4 - a^2}{2(a+1)(a+2)} = \frac{a^2 + 6a + 5}{2(a+1)(a+2)}$$

$$= \frac{(a+1)(a+5)}{2(a+1)(a+2)} = \frac{a+5}{2a+4}.$$

Blunders (-3)

B1 Numbers not consecutive

B2 Error in substitution

B3 Error in simplification

Slips (-1)

S1 Error in calculations

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20(10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a) **10 marks** **Att 3**

6. (a) In a certain subject, the examination consists of a project, a practical test, and a written paper. The overall mark is the weighted mean of the percentages achieved in these three components, using the weights 2, 3 and 5, respectively.

Michael scores 65% in the project and 80% in the practical.

What percentage mark must he get in the written paper in order to get an overall result of 70%?

(a) **10 marks** **Att 3**

6. (a) Let Michael require x% on written paper.

$$\therefore \text{ Weighed mean} = \frac{2(65)+3(80)+5(x)}{2+3+5} = \frac{370+5x}{10} = 70.$$

$$\therefore\ 370+5x = 700 \ \Rightarrow\ 5x = 330 \ \Rightarrow\ x = 66.$$

$\therefore$ 66% required on written paper.

Blunders (-3)

B1 Error in using weights

B2 Error in setting up the equation

B3 Error in solving equation

Slips (-1)

S1 Error in calculations

Attempts

A1 Tries to find the mean

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

6 (b) Solve the difference equation $u_{n+2} - 4u_{n+1} + u_n = 0,$ where $n \geq 0,$ given that $u_0 = 1$ and $u_1 = 2.$

(b) Form quadratic	**5 marks**	**Att 2**
Solve quadratic	**5 marks**	**Att 2**
General Term	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

6 (b)

$$u_{n+2} - 4u_{n+1} + u_n = 0.$$

$$\therefore x^2 - 4x + 1 = 0 \;\Rightarrow\; x = \frac{4 \pm \sqrt{16-4}}{2} \;\Rightarrow\; x = \frac{4 \pm 2\sqrt{3}}{2} \;\Rightarrow\; x = 2 \pm \sqrt{3}.$$

$$u_n = l(\alpha)^n + m(\beta)^n = l\left(2+\sqrt{3}\right)^n + m\left(2-\sqrt{3}\right)^n.$$

$$u_0 = 1 \;\Rightarrow\; l + m = 1 \text{ and } u_1 = 2 \;\Rightarrow\; l\left(2+\sqrt{3}\right) + m\left(2-\sqrt{3}\right) = 2.$$

$$\therefore\; 2(l+m) + \sqrt{3}(l-m) = 2 \;\Rightarrow\; 2 + \sqrt{3}(1-m-m) = 2$$

$$\Rightarrow\; \sqrt{3}(1-2m) = 0 \;\Rightarrow\; m = \frac{1}{2} \text{ and } l = \frac{1}{2}.$$

$$\therefore\; u_n = \frac{1}{2}\left[\left(2+\sqrt{3}\right)^n + \left(2-\sqrt{3}\right)^n\right].$$

Blunders (-3)

B1 Error in setting up quadratic
B2 Error in solving quadratic
B3 Error in finding General Term
B4 Error in finding *l* and *m*

Slips (-1)

S1 Error in calculations

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

6 **(c)** A bag contains discs of three different colours.
There are 5 red discs, 1 white disc and x black discs.
Three discs are picked together at random.

(i) Write down an expression in x for the probability that the three discs are all different in colour.

(ii) If the probability that the three discs are all different in colour is equal to the probability that they are all black, find x.

(c) (i) **10 marks** **Att 3**

6. (c) (i)

P(three discs different in colour)

$$= \frac{5 \times 1 \times x}{{}^{6+x}C_3} = \frac{5x}{\frac{(6+x)(5+x)(4+x)}{1 \times 2 \times 3}} = \frac{30x}{(6+x)(5+x)(4+x)}.$$

Blunders (-3)

B1 Error in numerator

B2 Error in denominator

(c)(ii) Black **5 marks** **Att 2**

Finish **5 marks** **Att 2**

6. (c) (ii)

P(three black discs)

$$= \frac{{}^{x}C_3}{{}^{6+x}C_3} = \frac{x(x-1)(x-2)}{(6+x)(5+x)(4+x)}.$$

$$\therefore \frac{x(x-1)(x-2)}{(6+x)(5+x)(4+x)} = \frac{30x}{(6+x)(5+x)(4+x)}.$$

$$\therefore (x-1)(x-2) = 30 \Rightarrow x^2 - 3x - 28 = 0.$$

$$\therefore (x-7)(x+4) = 0 \Rightarrow x = 7 \text{ as } x \neq -4.$$

$\therefore$ 7 black discs.

Blunders (-3)

B1 Error in total outcomes

B2 Error in total favourable

B3 Error in solving equation

Slips (-1)

S1 Error in calculations

QUESTION 7

Part (a)	**10 (5, 5) marks**	**Att (- , 2)**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (- , 2)**

7. **(a)** Katie must choose five subjects from nine available subjects.

The nine subjects include French and German.

(i) How many different combinations of five subjects are possible?

(ii) How many different combinations are possible if Katie wishes to study German but not French?

(a) (i) **5 marks** **Hit / Miss**

7. (a) (i) Number of combinations $= {}^{9}C_{5} = 126.$

(a) (ii) **5 marks** **Att 2**

7. (a) (ii) Number of combinations $= {}^{7}C_{4} = 35$

Blunders (-3)

B1 Error in *n* or *r*. (i.e., $n \neq 7$ or $r \neq 4$).

B2 Error in evaluation of ${}^{7}C_{4}$

Slips (-1)

S1 Error in calculations

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

7. (b) (b) Four cards are drawn together from a pack of 52 playing cards.
Find the probability that

(i) the four cards drawn are the four aces
(ii) two of the cards are clubs and the other two are diamonds
(iii) there are three clubs and two aces among the four cards.

(b)(i) **10 marks** **Att 3**

7. (b) (i)

$$\text{Probability (four aces)} = \frac{{}^{4}C_{4}}{{}^{52}C_{4}} = \frac{1}{270725}.$$

Blunders (-3)

B1 Incorrect total possible

B2 Incorrect total favourable

Slips (-1)

S1 Errors in calculations

(b) (ii) **5 marks** **Att 2**

7. (b) (ii) Probability (2 clubs and 2 diamonds) = $\frac{{}^{13}C_2 \times {}^{13}C_2}{{}^{52}C_4} = \frac{78 \times 78}{270725} = \frac{6084}{270725}$.

Blunders (-3)
B1 Incorrect total possible
B2 Incorrect total favourable

Slips (-1)
S1 Errors in calculations

(b) (iii) **5 marks** **Att 2**

7. (b) (iii) Probability $= \frac{1 \times {}^{12}C_2 \times {}^{3}C_1}{{}^{52}C_4} = \frac{198}{270725}$.

Blunders (-3)
B1 Incorrect total possible
B2 Incorrect total favourable

Slips (-1)
S1 Errors in calculations

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

7. (c) **(i)** The arithmetic mean of the three numbers x_1, x_2, x_3 is $\bar{x}$.

Let $d_1 = x_1 - \bar{x}$, $d_2 = x_2 - \bar{x}$ and $d_3 = x_3 - \bar{x}$.

Show that $\sum_{r=1}^{3} d_r = 0$.

(ii) The standard deviation of the three numbers x_1, x_2, x_3 is σ.

Given any real number b, let $k^2 = \sum_{r=1}^{3} \frac{(d_r - b)^2}{3}$.

Show that $\sigma^2 = k^2 - b^2$.

(i) Mean **5 marks** **Att 2**

Show Σ*d* = 0 **5 marks** **Att 2**

7. (c) (i)

$$\bar{x} = \frac{x_1 + x_2 + x_3}{3} \Rightarrow x_1 + x_2 + x_3 = 3\bar{x}.$$

$$\sum_{r=1}^{3} d_r = d_1 + d_2 + d_3 = x_1 - \bar{x} + x_2 - \bar{x} + x_3 - \bar{x} = x_1 + x_2 + x_3 - 3\bar{x} = 0.$$

Blunders (-3)

B1 Error in mean

B2 Error in evaluating $\sum_{r=1}^{3} d_r = 0$.

Expression for *k*^{2} **5 marks** **Att 2**

Finish **5 marks** **Att 2**

7. (c) (ii)

$$k^2 = \sum_{r=1}^{3} \frac{(d_r - b)^2}{3} = \frac{(d_1 - b)^2 + (d_2 - b)^2 + (d_3 - b)^2}{3}$$

$$= \frac{(x_1 - \bar{x} - b)^2 + (x_2 - \bar{x} - b)^2 + (x_3 - \bar{x} - b)^2}{3}$$

$$= \frac{(x_1 - \bar{x})^2 - 2b(x_1 - \bar{x}) + b^2 + (x_2 - \bar{x})^2 - 2b(x_2 - \bar{x}) + b^2 + (x_3 - \bar{x})^2 - 2b(x_3 - \bar{x}) + b^2}{3}$$

$$= \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2}{3} - \frac{2b[(x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 - \bar{x})]}{3} + b^2.$$

$$= \sigma^2 - 0 + b^2.$$

$$\therefore \ k^2 = \sigma^2 + b^2 \Rightarrow \sigma^2 = k^2 - b^2.$$

Blunders (-3)

B1 Error in handling $k^2 = \sum_{r=1}^{3} \frac{(d_r - b)^2}{3}$.

B2 Fails to finish

QUESTION 8

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

8. **(a)** Use the ratio test to show that $\sum_{n=1}^{\infty} \frac{2^{3n+1}}{n!}$ is convergent.

(a) u_{n+1}	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

8 (a)

$$\sum_{n=1}^{\infty} \frac{2^{3n+1}}{n!}. \qquad u_{n+1} = \frac{2^{3n+4}}{(n+1)!}.$$

$$\operatorname*{Limit}_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \operatorname*{Lim}_{n\to\infty}\left|\frac{2^{3n+4}}{(n+1)!}\times\frac{n!}{2^{3n+1}}\right| = \operatorname*{Lim}_{n\to\infty}\left|\frac{2^3}{n+1}\right| = 0 < 1. \quad \therefore \text{Convergent.}$$

Blunders (-3)

B1 Error in expressing $u_{n+1} = \frac{2^{3n+4}}{(n+1)!}$.

B2 Error in stating Ratio Test

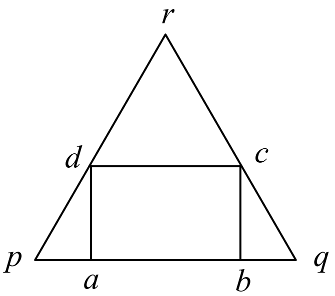
B3 Error in evaluating limit

Part (b) **20 (10, 10) marks** **Att (3, 3)**

8 (b) *pqr* is an equilateral triangle of side 6 cm.
abcd is a rectangle inscribed in the triangle as shown.
$|ab| = x$ cm and $|bc| = y$ cm.

(i) Express y in terms of x.

(ii) Find the maximum possible area of *abcd*.



(b) (i) **10 marks** **Att 3**

8 (b) (i)

$$|pq| = 6 \text{ and } |ab| = x \Rightarrow |bq| = 3 - \tfrac{1}{2}x.$$

$$|\angle cqb| = 60^\circ.\ \tan\angle cqb = \frac{|bc|}{bq} \Rightarrow \frac{y}{3-\frac{1}{2}x} = \tan 60^\circ.$$

$$\therefore\ y = \left(3 - \tfrac{1}{2}x\right)\sqrt{3} \text{ cm.}$$

Blunders (-3)

B1 Fails to express $|bq|$ in terms of x.

B2 Error in trig ratio or in use of similar triangles

B3 Error in setting up the equation

Slips (-1)

S1 Errors in calculations

(b) (ii) **10 marks** **Att 3**

8 (b) (ii)

$$\text{Area } abcd = A = xy = x\sqrt{3}\left(3 - \tfrac{1}{2}x\right).$$

$$A = 3\sqrt{3}x - \tfrac{1}{2}\sqrt{3}x^2.$$

$$\therefore\ \frac{dy}{dx} = 3\sqrt{3} - \sqrt{3}x = 0 \text{ for maximun area.} \quad \therefore\ x = 3.$$

$$\text{For } x = 3,\ \frac{d^2y}{dx^2} = -\sqrt{3} < 0 \Rightarrow \text{ maximum.}$$

$$A = 3\sqrt{3}\left(3 - \tfrac{3}{2}\right) = \frac{9\sqrt{3}}{2} \text{cm}^2.$$

* Incorrect y from part (i) ⇒ attempt at most for part (ii)

Blunders (-3)

B1 Error in expression for area

B2 Error in differentiation

B3 Error in solving equation

B4 Does not find area

Slips (-1)

S1 Errors in calculations

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

8 (c) (i) Derive the Maclaurin series for $f(x) = \cos x$, up to and including the term containing x^4.

(ii) Hence, or otherwise, show that the first three non-zero terms of the Maclaurin series for $f(x) = \cos^2 x$ are $1 - x^2 + \frac{x^4}{3}$.

(iii) Use these to find an approximation for $\cos^2(0.2)$, giving your answer correct to four decimal places.

(c) (i) **10 marks** **Att 3**

8 (c) (i)

$$f(x) = \cos x = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(x)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \text{.........}$$

$$f(0) = \cos 0 = 1.$$

$$f'(x) = -\sin x \quad \Rightarrow \quad f'(0) = -\sin 0 = 0.$$

$$f''(x) = -\cos x \quad \Rightarrow \quad f'''(0) = -\cos 0 = -1.$$

$$f'''(x) = \sin x \quad \Rightarrow \quad f'''(0) = \sin 0 = 0.$$

$$f^{iv}(x) = \cos x \quad \Rightarrow \quad f^{iv}(x) = \cos 0 = 1.$$

$$\therefore \; f(x) = \cos x = 1 + 0 - \frac{x^2}{2!} + 0 + \frac{x^4}{4!} \quad = \quad 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \; \text{.........}$$

Blunders (-3)

B1 Incorrect differentiation

B2 Incorrect evaluation of $f^{(n)}(0)$

B3 Each term not derived

B4 Error in Maclaurin Series

Slips (-1)

S1 Error in calculations

(c) (ii) **5 marks** **Att 2**

8 (c) (ii)

$$cos^2 x = \frac{1}{2}(1+\cos 2x) = \frac{1}{2}\left(1+1-\frac{4x^2}{2}+\frac{16x^4}{24}\right) = \frac{1}{2}\left(2-2x^2+\frac{2x^4}{3}\right).$$

$$\therefore \cos^2 x = 1-x^2+\frac{x^4}{3}.$$

Blunders (-3)
B1 Error in trig or multiplication

Slips (-1)
S1 Errors in calculations

(c) (iii) **5 marks** **Att 2**

8 (c) (iii)

$$\cos^2 x = 1-x^2+\frac{x^4}{3}.$$

$$\Rightarrow \cos^2(0.2) = 1-0.04+0.00053 = 0.96053 = 0.9605.$$

Blunders (-3)
B1 Error in terms

Slips (-1)
S1 Error in calculations

QUESTION 9

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

9(a)

20% of the items produced by a machine are defective. Four items are chosen at random. Find the probability that none of the chosen items is defective.

(a) **10 marks** **Att 3**

9 (a) Probability (one defective) $= p = \frac{1}{5}$; probability (one not defective) $= 1 - p = q = \frac{4}{5}$.

Probability (four not defective) $= {}^4C_4\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4 = \frac{256}{625}$.

Blunders (-3)

B1 Incorrect p or q.

B2 Error in Binomial

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

9 (b) Anne and Brendan play a game in which they take turns throwing a die. The first person to throw a six wins. Anne has the first throw.

(i) Find the probability that Anne wins on her second throw.

(ii) Find the probability that Anne wins on her first, second or third throw.

(iii) By finding the sum to infinity of a geometric series, or otherwise, find the probability that Anne wins the game.

(b) (i) **10 marks** **Att 3**

9 (b) (i)

Probability (Anne wins on second throw)

= P (Anne loses on 1st throw). P (Brendan loses on 1st throw).P (Anne wins on 2nd throw)

$= \frac{5}{6}.\frac{5}{6}.\frac{1}{6} = \frac{25}{216}$.

Blunders (-3)

B1 Any extra throw included or each incorrect prob

(b) (ii) **5 marks** **Att 2**

9 (b) (ii)

$$\text{Probability (Anne wins on 3}^{\text{rd}}\text{ throw)} = \frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{1}{6} = \frac{625}{7776}.$$

$$\text{Probability (Anne wins on 1}^{\text{st}}\text{, 2}^{\text{nd}}\text{ or 3}^{\text{rd}}\text{ throw)} = \frac{1}{6} + \frac{25}{216} + \frac{625}{7776} = \frac{2821}{7776}.$$

Blunders (-3)

B1 Each probability omitted

Slips (-1)

S1 Errors in calculations

(b) (iii) **5 marks** **Att 2**

9 (b) (iii)

$$p = \text{probability(Anne wins game)} = \frac{1}{6} + \frac{25}{216} + \frac{625}{7776} + \ldots\ldots\ldots\ldots$$

i.e. the sum to infinite of a geometric series where $a = \frac{1}{6}$ and $r = \frac{25}{36}$.

$$p = \frac{a}{1-r} = \frac{\frac{1}{6}}{1-\frac{25}{36}} = \frac{\frac{1}{6}}{\frac{11}{36}} = \frac{6}{11}.$$

Or

$p = P(\text{Anne eventually wins})$

$\quad = P(\text{person whose turn is next eventually wins})$

$$P(\text{Ann wins}) + P(\text{Brendan wins}) = 1$$

$$p + \frac{5}{6}p = 1$$

$$\frac{11}{6}p = 1$$

$$p = \frac{6}{11}$$

Blunders (-3)

B1 Error in a or r

B2 Error in sum to infinity

Slips (-1)

S1 Errors in calculations

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

9 (c)	In order to test the hypothesis that a particular coin is unbiased, the coin is tossed 400 times. The number of heads observed is x. Between what limits should x lie in order that the hypothesis not be rejected at the 5% significance level?

(c) Find μ	**5 marks**	**Att 2**
Find σ	**5 marks**	**Att 2**
Standard units	**5 marks**	**Att 2**
Conclusion	**5 marks**	**Att 2**

9 (c)

$$n = 400,\ p = \frac{1}{2},\ q = \frac{1}{2}.$$

$$\mu = np = 200 \text{ and } \sigma = \sqrt{npq} = 10.$$

$$-1.96 \le z \le 1.96 \Rightarrow -1.96 \le \frac{x-200}{10} \le 1.96.$$

$$\therefore -19.6 \le x - 200 \le 19.6 \Rightarrow 180.4 \le x \le 219.6.$$

$$\therefore 181 \le x \le 219.$$

Blunders (-3)

B1 Error in finding mean
B2 Error in finding standard deviation
B3 Error in units
B4 Error in conclusion

Slips (-1)

S1 Errors in calculations

QUESTION 10

Part (a)	**20 (5, 5, 10) marks**	**Att (- , - , 3)**
Part (b)	**30 (10, 10, 5, 5) marks**	**Att (3, 3, 2, 2)**

Part (a) **20 (5, 5, 10) marks** **Att (- , - , 3)**

10 (a) Let $x \oplus y = x + y - 4$, where $x, y \in \mathbf{Z}$.

(i) Find the identity element.

(ii) Find the inverse of x.

(iii) Determine whether $\oplus$ is associative on **Z**.

(a) (i) **5 marks** **Hit / miss**

10 (a) (i)

$$x \oplus e = x + e - 4 = x.$$
$$\therefore\ e = 4.$$

(a) (ii) **5 marks** **Hit / miss**

10 (a) (ii)

$$x \oplus x^{-1} = e = 4.$$
$$\therefore\ x + x^{-1} - 4 = 4 \ \Rightarrow\ x^{-1} = 8 - x.$$

(a) (iii) **10 marks** **Att 3**

10 (a) (iii)

If associative then: $(x \oplus y) \oplus z = x \oplus (y \oplus z)$.

$(x + y - 4) \oplus z = x \oplus (y + z - 4)$

$(x + y - 4) + z - 4 = x + (y + z - 4) - 4$

$x + y + z - 8 = x + y + z - 8$, as $+$ is associative on **Z**.

$\therefore$Operation $\oplus$ is associative.

Blunders (-3)

B1 Error in defining associativity.

B2 Error in applying rule

B3 No conclusion

Slips (-1)

S1 Error in calculations

Part (b) **30 (10, 10, 5, 5) marks** **Att (3, 3, 2, 2)**

10 (b) $(A, \circ)$ and $(B, *)$ are two groups. $A = \{k, l, m, n\}$ and $B = \{p, q, r, s\}$, and the Cayley tables for $(A, \circ)$ and $(B, *)$ are shown.

A:

$\circ$	k	l	m	n
k	l	k	n	m
l	k	l	m	n
m	n	m	k	l
n	m	n	l	k

B:

$*$	p	q	r	s
p	r	s	p	q
q	s	p	q	r
r	p	q	r	s
s	q	r	s	p

(i) Write down the identity element of $(A, \circ)$ and hence find a generator of $(A, \circ)$.

(ii) Find the order of each element in $(B, *)$.

(iii) Give an isomorphism ϕ from $(A, \circ)$ to $(B, *)$, justifying fully that it is an isomorphism.

(b) (i) **10 marks** **Att 3**

10 (b) (i)

l is the identity of $(A, \circ)$.

$m^1 = m,\ m^2 = k,\ m^3 = n,\ m^4 = l.\quad \therefore\ m$ is a generator. n is also a generator.

Blunders (-3)

B1 Error in selecting Identity

B2 Error in verifying generator

(b) (ii) **10 marks** **Att 3**

10 (b) (ii)

r is the identity of $(B, *)$. r is of order 1.

$p^2 = r \quad \Rightarrow \quad p$ is of order 2.

$q^4 = r \quad \Rightarrow \quad q$ is of order 4.

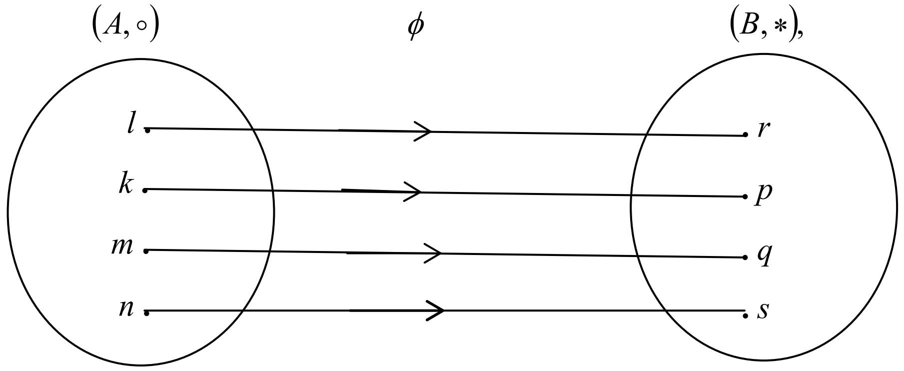
$s^4 = r \quad \Rightarrow \quad s$ is of order 4.

Blunders (-3)

B1 Error in order each time

(iii) Isomorphism	**5 marks**	**Att 2**
Justify	**5 marks**	**Att 2**

10 (b) (iii)



Justification:

l is the identity of $(A, \circ)$ and r is the identity of $(B, *)$.

Products involving the identity will clearly carry across. Others are:

$\phi(k \circ k) = \phi(l) = r$ and $\phi(k) * \phi(k) = p * p = r$.

$\phi(k \circ m) = \phi(n) = s$ and $\phi(k) * \varphi(m) = p * q = s$.

$\phi(k \circ n) = \phi(m) = q$ and $\phi(k) * \phi(n) = p * s = q$.

$\phi(m \circ m) = \phi(k) = p$ and $\phi(m) * \phi(m) = q * q = p$.

$\phi(m \circ k) = \phi(n) = s$ and $\phi(m) * \phi(k) = q * p = s$.

$\phi(m \circ n) = \phi(l) = r$ and $\phi(m) * \phi(n) = q * s = r$.

$\phi(n \circ n) = \phi(k) = p$ and $\phi(n) * \phi(n) = s * s = p$.

$\phi(n \circ k) = \phi(m) = q$ and $\phi(n) * \phi(k) = s * p = q$.

$\phi(n \circ m) = \phi(l) = r$ and $\phi(n) * \phi(m) = s * q = r$.

$\therefore \phi$ is an isomorphism.

* Note: the other possible isomorphism is: $l \to r$, $k \to p$, $m \to s$, $n \to q$.

Blunders (-3)

B1 Error in selecting isomorphism

B2 Not fully justified

QUESTION 11

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 marks** **Att 3**

11 (a) Find the coordinates of the point that is invariant under the transformation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}.$$

(a) **10 marks** **Att 3**

11 (a)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}. \ \therefore \ \begin{pmatrix} 2x+3y+5 \\ 4x-5y+2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

$$\therefore \ x+3y=-5 \ \Rightarrow \ 2x+6y=-10$$

$$4x-6y=-2 \ \Rightarrow \ \underline{4x-6y=-2}$$

$$6x=-12 \ \Rightarrow \ x=-2 \text{ and } y=-1.$$

$\therefore \ (-2,-1)$ is an invariant point.

Blunders (-3)

B1 Error in multiplication or addition of matrices

B2 Error in setting up equations

B3 Error in solving equations

Slips (-1)

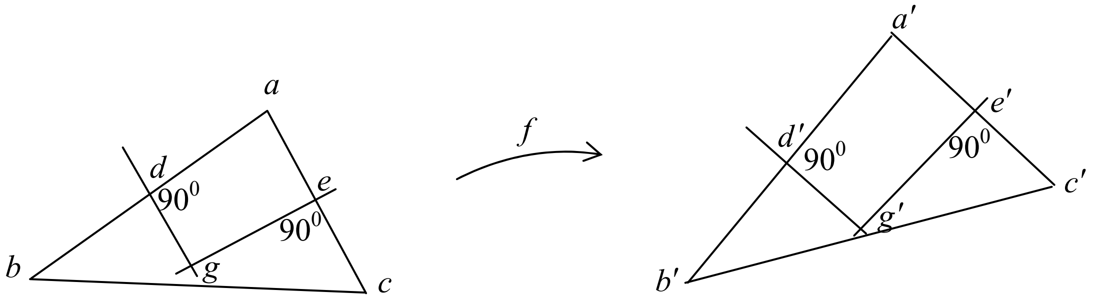
S1 Errors in calculations

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

11 (b) Prove that a similarity transformation maps the circumcentre of a triangle to the circumcentre of the image of the triangle.

(b) Circumcentre	**5 marks**	**Att 2**
Midpoints	**5 marks**	**Att 2**
Perpendicularity	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

11 (b)



dg and eg are the perpendicular bisectors of $[ab]$ and $[ac]$ respectively
$\therefore g$ is the circumcentre of $\Delta\, abc$.
d and e are the mid-points of $[ab]$ and $[ac]$ respectively $\Rightarrow$ d' and e' are the mid-points of $[a'b']$ and $[a'c']$ respectively, as mid-point is an invariant map.
$dg \perp ab$ and $eg \perp ac \Rightarrow d'g' \perp a'b'$ and $e'g' \perp a'c'$ as f is a similarity transformation.
$\therefore g'$ is the circumcentre of $\Delta\, a'b'c'$ and $f(g) = g'$.

Blunders (-3)

B1 Error in finding circumcentre
B2 Fails to identify mid points
B3 fails to identify perpendiculars
B4 No conclusion

Part (c) **20 (10, 10) marks** **Att (3, 3)**

11 (c)(i) E is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and f is the transformation

$$(x, y) \to (x', y'), \text{ where } x' = \frac{x}{a} \text{ and } y' = \frac{y}{b}.$$

Show that f maps E to the unit circle.

(ii) Hence, or otherwise, prove that the tangents to an ellipse at the endpoints of a diameter are parallel to each other.

(c) (i) **10 marks** **Att 3**

11 (c) (i)

$x = ax'$ and $y = by'$.

$$\therefore\ f(E): \frac{a^2x^2}{a^2}+\frac{b^2y^2}{b^2}=1 \Rightarrow x^2+y^2=1.$$

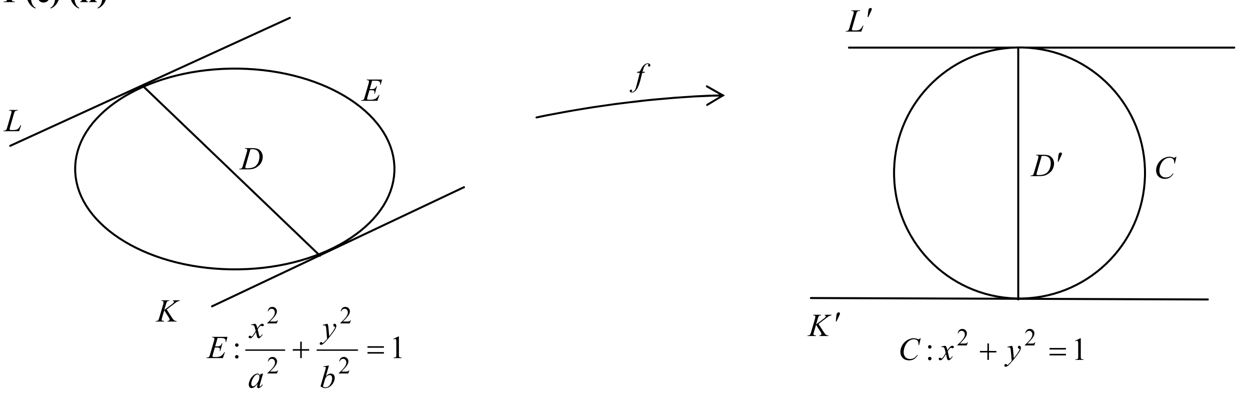
Blunders (-3)

B1 Error in finding images

B2 Error in finding equation of circle

(c) (ii) **10 marks** **Att 3**

11 (c) (ii)



$E: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\qquad$ $C: x^2+y^2=1$

By f, E maps to C and L, K, D map onto L', K', D' respectively.
But $L' \perp D'$ and $K' \perp D'$ as tangent to a circle is perpendicular to diameter at point of contact.

$\therefore$ L' is parallel to K'.

$\therefore$ $f^{-1}(L)$ is parallel to $f^{-1}(K')$ as parallelism is invariant.

$\therefore$ L is parallel to K.

Blunders (-3)

B1 Fails to define tangent to circle

B2 Fails to mention invariance of parallel lines

MARCANNA BREISE AS UCHT FREAGAIRT TRÍ GHAEILGE

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc × 5% = 9·9 ⇒ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] × 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0