

Scoil Mhuire V[14-15] Problem Set 4

1. Factorise the following quadratics: (i) $4x^2 + 14x + 10$ (ii) $3x^2 - 21x - 24$ (iii) $5p^2 + 4pq - q^2$

2. Given that k is real, find the set of value of k for which the equation

$$(1 + 2k)x^2 - 10x + (k - 2) = 0 \text{ does not have real roots.}$$

3. Solve the following equations:

$$(i) \log_3(10x + 7) - \log_3(x + 1) = 2 \quad (ii) \ln(4x + 1) = 2.5649 \quad (iii) 4^x = 8 \quad (iv) e^{2x+4} = 0.1353$$

4. *Let $f(x) = x^3 + kx^2 - 4x - 12$, where k is a constant. Given that $x + 3$ is a factor of $f(x)$, find k .

5. *Solve the inequality $\frac{x+1}{x-1} < 4$, where $x \in R$ and $x \neq 1$

6. Solve the equation $3^{x+1} + 3^{1-x} = 10$

7. *The cubic equation $4x^3 + 10x^2 - 7x - 3 = 0$ has one integer root and two irrational roots.

Find all roots and express the irrational roots in simplest surd form.

8. Write down a cubic equation that has roots of -1, 4 and -2 in the form of $ax^3 + bx^2 + cx + d = 0$ where $a, b, c \in Z$.

9. Given that $f(x) = x^2 + 6x + 2$ can be written as $(x + a)^2 + b$

(i) Using the method of completing the square, find the values of a and b .

(ii) Hence, find the co-ordinates of the local minimum of the curve.

(iii) Solve the equation $f(x) = 0$, writing your answers in surd form.

10. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400e^{-0.05t} + 25, t \geq 0.$$

(i) Find the temperature of the ball as it enters the liquid.

(ii) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(iii) From the equation for temperature T in terms of t , given above, explain why the temperature of the ball can never fall to 25 °C.

Answers: 2. $-3 < x < \frac{9}{2}$ 3.(i) 2 (ii) 3 (iii) 1.5 (iv) -3 4. $k = 3$ 5. $\left[x > \frac{5}{3} \text{ and } x < 1 \right]$ 6. $[1, -1]$

7. $\left[x = -3, x = \frac{1 \pm \sqrt{5}}{4} \right]$ 8. $x^3 - x^2 - 10x - 8$ 9. (i) $a = 3, b = -7$ (ii) $(-3, -7)$ (iii) $x = -3 \pm \sqrt{7}$

10. (i) 25 °C (ii) 7.49