

TY Hons Maths: Problem Set 3

1. Factorise fully the following:

(i) $x^2 + 2xy + y^2$ (ii) $x^2 - 2xy + y^2$ (iii) $4k^2 + 7kt - 2t^2$ (iv) $x^4 - y^4$

2. Simplify $\frac{x^{-\frac{1}{2}} + x^{\frac{1}{2}}}{x^{-\frac{1}{2}} - x^{\frac{3}{2}}}$

3. Prove that $k + 1$ is a root of the function $f(x) = x^2 - 2x - k^2 + 1$, where k is a constant.

4. A quadratic function has roots of -2 and -1. It also contains the point (-3, 8).
Evaluate the function in the form $ax^2 + bx + c = 0$ where $a, b, c \in \mathbb{Z}$

5. Find the real numbers a and b such that

$$2x^2 + 6x + 4 = k(x + r)^2 + p$$

6. The area of a rectangle, $A(x) = 6x^2 + 4x - 2$. If the length is given by $(3x - 1)$, find
(i) an expression for the width of the rectangle.
(ii) an expression for the perimeter, $P(x)$, of the rectangle.

7. Write down a quadratic equation that has roots of $\frac{1}{2}$ and $\frac{1}{4}$ in the form of $ax^2 + bx + c = 0$
where a, b and $c \in \mathbb{Z}$

8. Solve the following systems of equations:

$$\begin{aligned}x + y + z &= 6 \\x - 2y - z &= -2 \\3x - 5y + z &= 0\end{aligned}$$

