



**Coimisiún na Scrúduithe Stáit
State Examinations Commission**

LEAVING CERTIFICATE EXAMINATION, 2006

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

THURSDAY, 8 JUNE – MORNING, 9:30 to 12:00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

**Answers should include the appropriate units of measurement,
where relevant.**

1. (a) Find the real number a such that for all $x \neq 9$,

$$\frac{x-9}{\sqrt{x}-3} = \sqrt{x} + a.$$

- (b) $f(x) = 3x^3 + mx^2 - 17x + n$, where m and n are constants.
Given that $x-3$ and $x+2$ are factors of $f(x)$, find the value of m and the value of n .

- (c) $x^2 - t$ is a factor of $x^3 - px^2 - qx + r$.

(i) Show that $pq = r$.

(ii) Express the roots of $x^3 - px^2 - qx + r = 0$ in terms of p and q .

2. (a) Solve the simultaneous equations

$$y = 2x - 5$$

$$x^2 + xy = 2.$$

- (b) (i) Find the range of values of $t \in \mathbf{R}$ for which the quadratic equation $(2t-1)x^2 + 5tx + 2t = 0$ has real roots.

(ii) Explain why the roots are real when t is an integer.

- (c) $f(x) = 1 - b^{2x}$ and $g(x) = b^{1+2x}$, where b is a positive real number.

Find, in terms of b , the value of x for which $f(x) = g(x)$.

3. (a) Given that $z = 2 + i$, where $i^2 = -1$, find the real number d such that $z + \frac{d}{z}$ is real.

- (b) (i) Use matrix methods to solve the simultaneous equations

$$4x - 2y = 5$$

$$8x + 3y = -4$$

- (ii) Find the two values of k which satisfy the matrix equation

$$(1 \quad k) \begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ k \end{pmatrix} = 11.$$

- (c) (i) Express $-8 - 8\sqrt{3}i$ in the form $r(\cos\theta + i\sin\theta)$.

- (ii) Hence find $(-8 - 8\sqrt{3}i)^3$.

- (iii) Find the four complex numbers z such that

$$z^4 = -8 - 8\sqrt{3}i.$$

Give your answers in the form $a + bi$, with a and b fully evaluated.

4. (a) $-2 + 2 + 6 + \dots + (4n - 6)$ are the first n terms of an arithmetic series. S_n , the sum of these n terms, is 160. Find the value of n .

- (b) The sum to infinity of a geometric series is $\frac{9}{2}$.

The second term of the series is -2 .

Find the value of r , the common ratio of the series.

- (c) The sequence u_1, u_2, u_3, \dots , defined by $u_1 = 3$ and $u_{n+1} = 2u_n + 3$, is as follows:
3, 9, 21, 45, 93, ...

- (i) Find u_6 , and verify that it is equal to the sum of the first six terms of a geometric series with first term 3 and common ratio 2.

- (ii) Given that, for all k , u_k is the sum of the first k terms of a geometric series

with first term 3 and common ratio 2, find $\sum_{k=1}^n u_k$.

5. (a) Find the value of the middle term of the binomial expansion of

$$\left(\frac{x}{y} - \frac{y}{x}\right)^8.$$

(b) (i) Express $\frac{2}{(r+1)(r+3)}$ in the form $\frac{A}{r+1} + \frac{B}{r+3}$.

(ii) Hence find $\sum_{r=1}^n \frac{2}{(r+1)(r+3)}$.

(iii) Hence evaluate $\sum_{r=1}^{\infty} \frac{2}{(r+1)(r+3)}$.

- (c) (i) Given two real numbers a and b , where $a > 1$ and $b > 1$, prove that

$$\frac{1}{\log_b a} + \frac{1}{\log_a b} \geq 2.$$

(ii) Under what condition is $\frac{1}{\log_b a} + \frac{1}{\log_a b} = 2$.

6. (a) Differentiate $\sqrt{x}(x+2)$ with respect to x .

(b) The equation of a curve is $y = 3x^4 - 2x^3 - 9x^2 + 8$.

- (i) Show that the curve has a local maximum at the point $(0, 8)$.

- (ii) Find the coordinates of the two local minimum points on the curve.

- (iii) Draw a sketch of the curve.

(c) Prove by induction that $\frac{d}{dx}(x^n) = nx^{n-1}$, $n \geq 1$, $n \in \mathbf{N}$.

7. (a) Taking $x_1 = 2$ as the first approximation to the real root of the equation $x^3 + x - 9 = 0$, use the Newton-Raphson method to find x_2 , the second approximation.

- (b) The parametric equations of a curve are:

$$x = 3\cos\theta - \cos^3\theta$$

$$y = 3\sin\theta - \sin^3\theta, \text{ where } 0 < \theta < \frac{\pi}{2}.$$

(i) Find $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$.

(ii) Hence show that $\frac{dy}{dx} = \frac{-1}{\tan^3\theta}$.

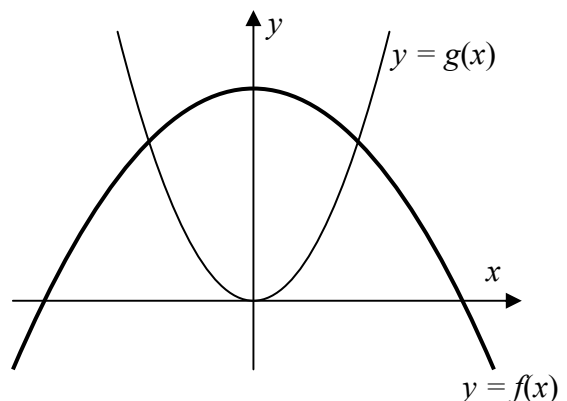
- (c) Given $y = \ln\left(\frac{3+x}{\sqrt{9-x^2}}\right)$, find $\frac{dy}{dx}$ and express it in the form $\frac{a}{b-x^n}$.

8. (a) Find (i) $\int \sqrt{x} dx$ (ii) $\int e^{-2x} dx$.

- (b) Evaluate (i) $\int_1^2 x(1+x^2)^3 dx$ (ii) $\int_0^{\frac{\pi}{4}} \sin 5\theta \cos 3\theta d\theta$.

- (c) The diagram shows the graphs of the curves $y = f(x)$ and $y = g(x)$, where $f(x) = 12 - 3x^2$ and $g(x) = 9x^2$.

- (i) Calculate the area of the region enclosed by the curve $y = f(x)$ and the x -axis.
- (ii) Show that the region enclosed by the curves $y = f(x)$ and $y = g(x)$ has half that area.



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