Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2007

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

THURSDAY, 7 JUNE – MORNING, 9:30 to 12:00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: **Marks will be lost if all necessary work is not clearly shown.**

Answers should include the appropriate units of measurement, where relevant.

1. (a) Simplify $\dfrac{x^2 - xy}{x^2 - y^2}$.

(b) Let $f(x) = x^2 + (k+1)x - k - 2$, where k is a constant.

(i) Find the value of k for which $f(x) = 0$ has equal roots.

(ii) Find, in terms of k, the roots of $f(x) = 0$.

(iii) Find the range of values of k for which both roots are positive.

(c) $x + p$ is a factor of both $ax^2 + b$ and $ax^2 + bx - ac$.

(i) Show that $p^2 = -\dfrac{b}{a}$ and that $p = \dfrac{-b - ac}{b}$.

(ii) Hence show that $p^2 + p^3 = c$.

2. (a) Solve the simultaneous equations

$$x + y + z = 2$$
$$2x + y + z = 3$$
$$x - 2y + 2z = 15.$$

(b) α and β are the roots of the equation $x^2 - 4x + 6 = 0$.

(i) Find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$.

(ii) Find the quadratic equation whose roots are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

(c) (i) Prove that $x + \dfrac{9}{x+2} \geq 4$, where $x + 2 > 0$.

(ii) Prove that $x + \dfrac{9}{x+a} \geq 6 - a$, where $x + a > 0$.

3. **(a)** Let $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 3 & \frac{3}{2} \end{pmatrix}$. Find $A^2 - 2A$.

(b) Let $z = -1 + i$, where $i^2 = -1$.

(i) Use De Moivre's theorem to evaluate z^5 and z^9.

(ii) Show that $z^5 + z^9 = 12z$.

(c) **(i)** Find the two complex numbers $a + bi$ for which $(a+bi)^2 = 15 + 8i$.

(ii) Solve the equation $iz^2 + (2 - 3i)z + (-5 + 5i) = 0$.

4. **(a)** Show that $\binom{n}{1} + \binom{n}{2} = \binom{n+1}{2}$ for all natural numbers $n \geq 2$.

(b) $u_1 = 5$ and $u_{n+1} = \frac{n}{n+1} u_n$ for all $n \geq 1, n \in \mathbf{N}$.

(i) Write down the value of each of u_2, u_3, and u_4.

(ii) Hence, by inspection, write an expression for u_n in terms of n.

(iii) Use induction to justify your answer for part **(ii)**.

(c) The sum of the first n terms of a series is given by $S_n = n^2 \log_e 3$.

(i) Find the n^{th} term and prove that the series is arithmetic.

(ii) How many of the terms of the series are less than $12 \log_e 27$?

5. **(a)** Plot, on the number line, the values of x that satisfy the inequality

$$|x+1| \leq 2, \text{ where } x \in \mathbf{Z}.$$

(b) In the expansion of $\left(2x - \frac{1}{x^2}\right)^9$,

(i) find the general term

(ii) find the value of the term independent of x.

(c) The n^{th} term of a series is given by nx^n, where $|x| < 1$.

(i) Find an expression for S_n, the sum of the first n terms of the series.

(ii) Hence, find the sum to infinity of the series.

6. **(a)** Differentiate $\frac{x^2 - 1}{x^2 + 1}$ with respect to x.

(b) **(i)** Differentiate $\frac{1}{x}$ with respect to x from first principles.

(ii) Find the equation of the tangent to $y = \frac{1}{x}$ at the point $\left(2, \frac{1}{2}\right)$.

(c) Let $f(x) = \tan^{-1}\frac{x}{2}$ and $g(x) = \tan^{-1}\frac{2}{x}$, for $x > 0$.

(i) Find $f'(x)$ and $g'(x)$.

(ii) Hence, show that $f(x) + g(x)$ is constant.

(iii) Find the value of $f(x) + g(x)$.

7. **(a)** Taking 1 as the first approximation of a root of $x^3 + 2x - 4 = 0$, use the Newton-Raphson method to calculate the second approximation of this root.

(b) **(i)** Find the equation of the tangent to the curve
$3x^2 + y^2 = 28$ at the point $(2, -4)$.

(ii) $x = e^t \cos t$ and $y = e^t \sin t$. Show that $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$.

(c) $f(x) = \log_e 3x - 3x$, where $x > 0$.

(i) Show that $(\frac{1}{3}, -1)$ is a local maximum point of $f(x)$.

(ii) Deduce that the graph of $f(x)$ does not intersect the x-axis.

8. **(a)** Find **(i)** $\int x^3 dx$ **(ii)** $\int \frac{1}{x^3} dx$.

(b) **(i)** Evaluate $\int_0^4 x\sqrt{x^2+9}\, dx$.

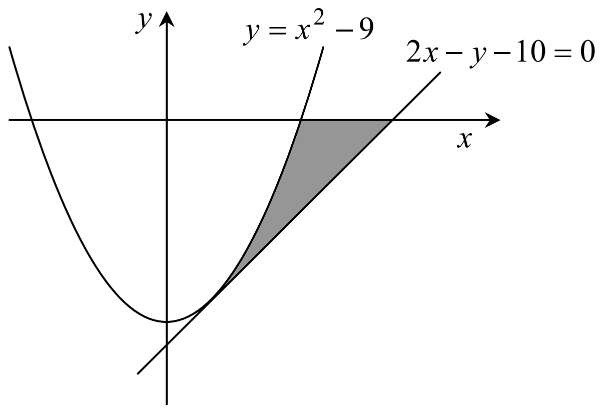
(ii) f is a function such that $f'(x) = 6 - \sin x$ and $f\left(\frac{\pi}{3}\right) = 2\pi$.

Find $f(x)$.

(c) The line $2x - y - 10 = 0$ is a tangent to the curve $y = x^2 - 9$, as shown.

The shaded region is bounded by the line, the curve and the x-axis.

Calculate the area of this region.



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