



**Coimisiún na Scrúduithe Stáit
State Examinations Commission**

LEAVING CERTIFICATE EXAMINATION, 2008

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

FRIDAY, 6 JUNE – MORNING, 9:30 to 12:00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

**Answers should include the appropriate units of measurement,
where relevant.**

1. (a) Simplify fully $\frac{x^2 + 4}{x^2 - 4} - \frac{x}{x + 2}$.

(b) Given that one of the roots is an integer, solve the equation

$$6x^3 - 29x^2 + 36x - 9 = 0.$$

(c) Two of the roots of the equation $ax^3 + bx^2 + cx + d = 0$ are p and $-p$. Show that $bc = ad$.

2. (a) Express $x^2 + 10x + 32$ in the form $(x + a)^2 + b$.

(b) α and β are the roots of the equation $x^2 - 7x + 1 = 0$.

(i) Find the value of $\alpha^2 + \beta^2$.

(ii) Find the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$.

(c) Show that if a and b are non-zero real numbers, then the value of

$\frac{a}{b} + \frac{b}{a}$ can never lie between -2 and 2 .

Hint: consider the case where a and b have the same sign separately from the case where a and b have opposite sign.

3. (a) Let A be the matrix $\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$.
Find the matrix B , such that $AB = \begin{pmatrix} 4 & 6 \\ 3 & 2 \end{pmatrix}$.

- (b) (i) Let $z = \frac{5}{2+i} - 1$, where $i^2 = -1$.
Express z in the form $a + bi$ and plot it on an Argand diagram.
- (ii) Use De Moivre's theorem to evaluate z^6 .

- (c) Prove, by induction, that
 $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ for $n \in \mathbb{N}$.

4. (a) $2 + \frac{2}{3} + \frac{2}{9} + \dots$ is a geometric series.
Find the sum to infinity of the series.

- (b) Given that $u_n = 2\left(-\frac{1}{2}\right)^n - 2$ for all $n \in \mathbb{N}$,

(i) write down u_{n+1} and u_{n+2}

(ii) show that $2u_{n+2} - u_{n+1} - u_n = 0$.

- (c) (i) Write down an expression in n for the sum $1 + 2 + 3 + \dots + n$
and an expression in n for the sum $1^2 + 2^2 + 3^2 + \dots + n^2$.

(ii) Find, in terms of n , the sum $\sum_{r=1}^n (6r^2 + 2r + 5 + 2^r)$.

5. (a) Find the range of values of x that satisfy the inequality

$$x^2 - 3x - 10 \leq 0.$$

- (b) (i) Solve the equation

$$2^{x^2} = 8^{2x+9}.$$

- (ii) Solve the equation

$$\log_e(2x+3) + \log_e(x-2) = 2 \log_e(x+4).$$

- (c) Show that there are no natural numbers n and r for which

$\binom{n}{r-1}$, $\binom{n}{r}$ and $\binom{n}{r+1}$ are consecutive terms in a geometric sequence.

6. (a) Differentiate $\sqrt{x^3}$ with respect to x .

- (b) Let $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

Show that $\frac{dy}{dx} = \frac{4}{(e^x + e^{-x})^2}$.

- (c) The function $f(x) = 2x^3 + 3x^2 + bx + c$ has a local maximum at $x = -2$.

- (i) Find the value of b .

- (ii) Find the range of values of c for which $f(x) = 0$ has three distinct real roots.

7. (a) Differentiate $2x + \sin 2x$ with respect to x .

(b) The equation of a curve is $5x^2 + 5y^2 + 6xy = 16$.

(i) Find $\frac{dy}{dx}$ in terms of x and y .

(ii) $(1, 1)$ and $(2, -2)$ are two points on the curve.

Show that the tangents at these points are perpendicular to each other.

(c) Let $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$.

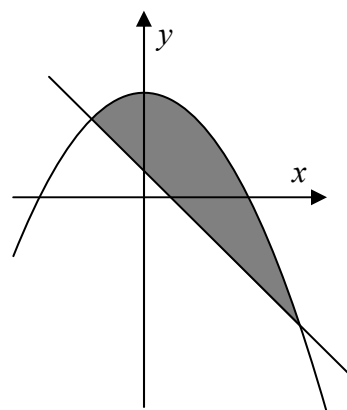
Find $\frac{dy}{dx}$ and express it in the form $\frac{a}{a+x^b}$, where $a, b \in \mathbf{N}$.

8. (a) Find $\int (2x + \cos 3x) dx$.

(b) Evaluate (i) $\int_0^1 3x^2 e^{x^3} dx$ (ii) $\int_2^4 \frac{2x^3}{x^2-1} dx$.

(c) The diagram shows the curve $y = 4 - x^2$ and the line $2x + y - 1 = 0$.

Calculate the area of the shaded region enclosed by the curve and the line.



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