

Problem Set 10 - Solutions

1 (i) For real roots, $b^2 - 4ac \geq 0$

$$a = 1+2k, \quad b = -10, \quad c = k-2$$

$$b^2 - 4ac \geq 0$$

$$(-10)^2 - 4(1+2k)(k-2) \geq 0$$

$$100 - 4(k-2+2k^2-4k) \geq 0$$

$$100 - 4k + 8 - 8k^2 + 16k \geq 0$$

$$-8k^2 + 12k + 108 \geq 0$$

$$8k^2 - 12k - 108 \geq 0$$

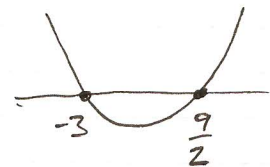
$$2k^2 - 3k - 27 \geq 0$$

$$k \leq -3 \text{ and } k \geq \frac{9}{2}$$

$$2k^2 - 3k - 27 = 0$$

$$(2k-9)(k+3) = 0$$

$$k = \frac{9}{2} \quad | \quad k = -3$$



(ii) $2 \log y = \log 2 + \log x$

$$\log y^2 = \log 2x$$

$$y^2 = 2x$$

$$y^2 = y$$

$$y^2 - y = 0$$

$$y(y-1) = 0$$

$$y = 0 \quad | \quad y = 1$$

$$x = 0 \quad | \quad x = \frac{1}{2}$$

$$2^y = 4^x$$

$$2^y = (2^2)^x$$

$$2^y = 2^{2x}$$

$$y = 2x$$

(iii) $2^{2x+1} - 15(2^x) = 8$

$$2u^2 - 15u - 8 = 0$$

$$(2u+1)(u-8) = 0$$

$$u = -\frac{1}{2} \quad | \quad u = 8$$

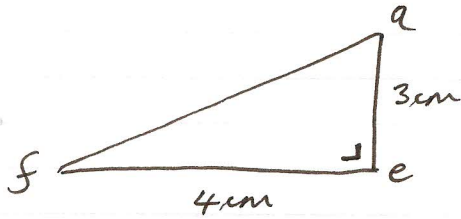
$$2^x = -2^{-1} \quad | \quad 2^x = 2^3$$

No soln

$$x = 3$$

$$\begin{aligned} 2^{2x+1} &= 2^{2x} \cdot 2 \\ &= 2 \cdot (2^x)^2 \\ &= 2u^2 \end{aligned}$$

2 // (i)



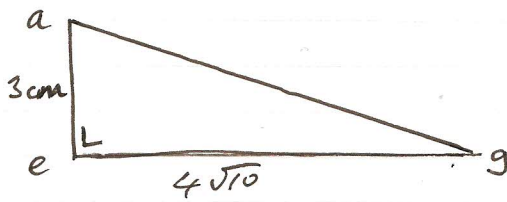
$$|af|^2 = |ae|^2 + |ef|^2$$

$$|af|^2 = 3^2 + 4^2$$

$$|af|^2 = 25$$

$$\Rightarrow |af| = 5$$

(ii)



$$|eg|^2 = |ef|^2 + |fg|^2$$

$$= 4^2 + 12^2$$

$$|eg|^2 = 16 + 144 = 160$$

$$|eg| = \sqrt{160} = 4\sqrt{10}$$

$$|ag|^2 = |ae|^2 + |eg|^2$$

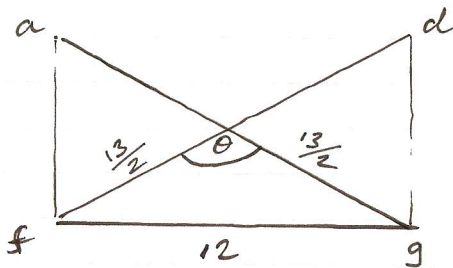
$$= 3^2 + (4\sqrt{10})^2$$

$$= 9 + 160$$

$$|ag|^2 = 169 \Rightarrow |ag| = \sqrt{169} = 13$$

(iii)

$|ag| = |df|$ and diagonals bisect each other.



Find θ

$$12^2 = \left(\frac{13}{2}\right)^2 + \left(\frac{13}{2}\right)^2 - 2\left(\frac{13}{2}\right)\left(\frac{13}{2}\right)\cos\theta$$

$$\cos\theta = -0.7041$$

$$\theta = \cos^{-1}(-0.7041)$$

$$\theta = 134.76$$

$$\Rightarrow \text{Acute Angle} = 180 - 134.76$$

$$= 45.24$$

$$\approx 45^\circ$$

3

$$\begin{aligned}
 f(x) &= a \left[x^2 - (\text{Sum of roots})x + (\text{Prod. of roots}) \right] \\
 &= a \left[x^2 - (-1)x + (-6) \right] \\
 &= a (x^2 + x - 6)
 \end{aligned}$$

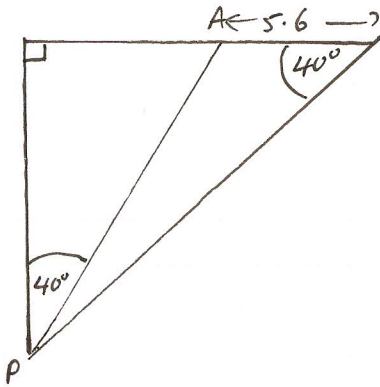
$$(1, -12) \Rightarrow f(1) = a(1^2 + 1 - 6) = -12$$

$$\Rightarrow -4a = -12$$

$$a = 3$$

$$f(x) = 3x^2 + 3x - 18$$

4



$$\frac{|AP|}{\sin 40^\circ} = \frac{5.6}{\sin 10^\circ}$$

$$|AP| = \frac{5.6(\sin 40^\circ)}{\sin 10^\circ}$$

$$|AP| = 20.73$$

$$\cos 40^\circ = \frac{|TP|}{20.73}$$

$$|TP| = 20.73(\cos 40^\circ)$$

$$= 15.8 \text{ m}$$

5

$$(i) z^3 - 1 = (z-1)(z^2 + z + 1)$$

$$(ii) z^2 + z + 1$$

$$a=1, b=1, c=1$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{3} \sqrt{-1}}{2}$$

$$= \frac{-1 \pm \sqrt{3} i}{2}$$

$$\text{Roots: } z = 1$$

$$z = -1 + \sqrt{3} i$$

$$z = -1 - \sqrt{3} i$$

