

## Scoil Mhuire V – Hons Maths 17/18

### Problem Set 11 – For Friday 24<sup>th</sup> March

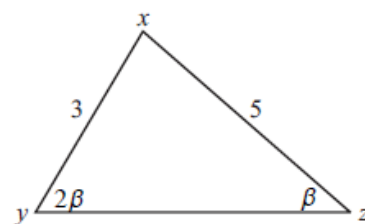
- The cubic equation  $x^3 + 4x^2 + x - 26 = 0$  has one integer root and two complex roots. Find all the roots.
- Show using the method of completing the square that  $2x^2 + 5x - 7$  can be expressed as  $k\left(x + \frac{a}{b}\right)^2 - \frac{c}{d}$ , where  $k, a, b, c, d \in \mathbb{Z}$ .
- The area of an equilateral triangle is  $4\sqrt{3} \text{ cm}^2$ . Find the length of a side of the triangle.

- In the triangle  $xyz$ ,  $|\angle xyz| = 2\beta$  and  $|\angle xzy| = \beta$ ,  $|xy| = 3$  and  $|xz| = 5$ .

(i) Use this information to express  $\sin 2\beta$  in the form  $\frac{a}{b} \sin \beta$

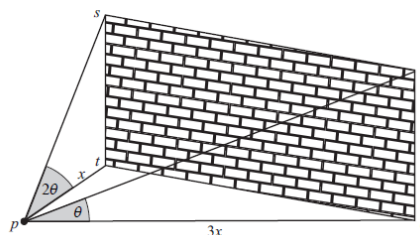
where  $a, b, c, d \in \mathbb{N}$

(ii) Hence express  $\tan \beta$  in the form  $\frac{\sqrt{c}}{d}$  where  $c, d \in \mathbb{N}$ .



- The diagram shows a vertical rectangular wall  $QRST$  of height  $h$  on level ground.  $P$  is a point on the ground in front of the wall. The angle of elevation of  $R$  from  $P$  is  $\theta$  and the angle of elevation of  $S$  from  $P$  is  $2\theta$ . Also it is given that  $|PQ| = 3|PT|$ .

- Express  $h$  in terms of  $\tan \theta$  and  $x$ .
- Express  $h$  in terms of  $\tan 2\theta$  and  $x$ .
- Hence evaluate  $\theta$ .



- The graph of  $f(x) = ax^3 + bx^2 + cx + d$  crosses the  $x$ -axis at  $x = 1$ ,  $x = -2$  and  $x = \frac{1}{2}$ . It also crosses the  $y$ -axis at the point  $(0,6)$ . Find the coefficients  $a, b, c$  and  $d$ .

- Simplify the following expressions giving your answer in the form  $a + bi$  where  $a, b \in \mathbb{R}$ .

$$(i) \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \left( \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)^4 \quad (ii) \frac{\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}}{\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}}$$

- In the given diagram, the line  $PT$  makes an angle of  $63.43^\circ$  with the  $x$ -axis at the point  $P(-5,0)$ . The line  $PT$  intersects the  $y$ -axis at  $S$  and  $|PS| = |ST|$ . The point  $R$  is on the  $x$ -axis such that  $|PO| : |OR| = 2 : 3$ . Find (i) the slope of  $PT$  to the nearest integer.

- the equation of  $PT$  in the form  $y = mx + c$
- the distance  $PS$  in surd form.
- the co-ordinates of  $T$ .
- the co-ordinates of  $R$ .
- the area of the triangle  $PTR$

