5th Year Hons Maths - Problem Set 6 - Solutions

1// No Real Roots $\Rightarrow b^2 - 4ac < 0$

$a = 3,\ b = 2,\ c = k$

$(2)^2 - 4(3)(k) < 0$

$4 - 12k < 0$

$12k > 4$

$k > \frac{1}{3}$

2// $V(t) = 965 - 86t$

(i) $V(4) = 965 - 86(4)$

$=$

$V(4)$ = Value of photocopier after 4 years

(ii) $V(t) = 578$

$965 - 86t = 578$

$86t = 387$

$t = 4.5$ years

$V(4.5) = 578$ is the value of the photocopier after 4.5 years

(iii) original Price $\Rightarrow t = 0$

$V(0) = 965 - 86(0) = 965$

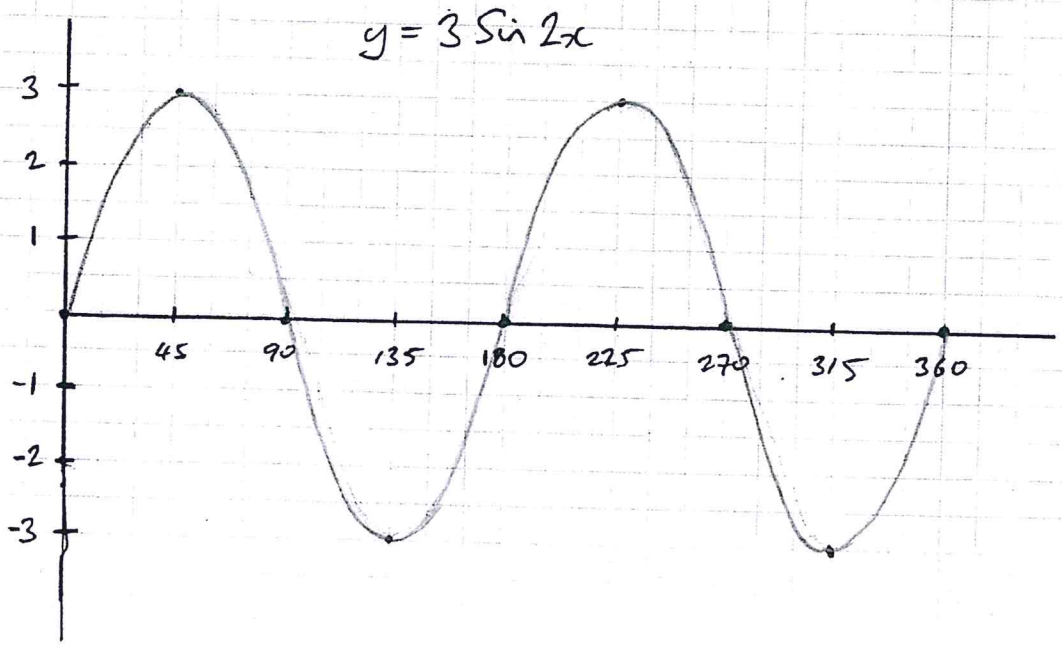
3// $2x = 90^\circ \Rightarrow x = 45^\circ$

x	0	45	90	135	180	225	270
$2x$	0	90	180	270	360	450	540
$\sin 2x$	0	1	0	-1	0	1	0
$3 \sin 2x$	0	3	0	-3	0	3	0

(0,0) (45,3) (90,0) (135,-3) (180,0)

(225,3) (270,0) (315,-3) etc

$y = 3 \sin 2x$



Period = 180°

Range = $[3, -3]$

Theory: $y = A \sin bx$

Period = $\frac{2\pi}{b}$

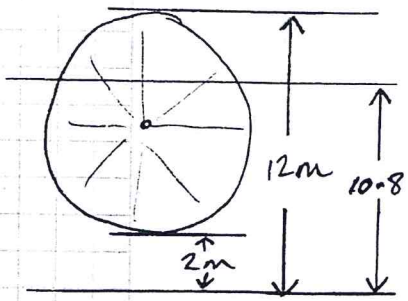
Range = $[A, -A]$

4 (i) Max value of $\sin t^\circ = 1$

$\Rightarrow$ Max height $h = 7 + 5(1) = 12\,m$

Min value of $\sin t^\circ = -1$

$\Rightarrow$ Min height: $h = 7 + 5(-1) = 2\,m$



(ii) $h = 7 + 5\sin t^\circ$

$10.8 = 7 + 5\sin t^\circ$

$3.8 = 5\sin t^\circ$

$\sin t^\circ = 0.76$

$\Rightarrow t^\circ = \sin^{-1}(0.76)$

$t = 49.64\,s$

What other value of t gives $\sin t^\circ = 0.76$

Ans: $180^\circ - 49.64^\circ$

$= 130.36\,s$

$\underline{t = 49.64}$ and $\underline{t = 130.36}$

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5 (i) $\log_2(10x+7) - \log_2(x+1) = 1$

$\log_2\left(\frac{10x+7}{x+1}\right) = 1$

$\frac{10x+7}{x+1} = 2^1$

$10x + 7 = 2(x+1)$

$8x = -5$

$x = \frac{-5}{8}$

(ii) $\ln(4x+1) = 1.0986$

$e^{\ln(4x+1)} = e^{1.0986}$

$4x + 1 = 3$

$x = \frac{1}{2}$

(iii) $e^{2x+4} = 7.389$

$\ln\left(e^{2x+4}\right) = \ln(7.389)$

$2x + 4 = 2$

$x = -1$

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6 TSA $= 2x^2 + 4x(x+1)$ [4 sides have equal area of $x(x+1)$]

$= 2x^2 + 4x^2 + 4x$

TSA $= 6x^2 + 4x$

$TSA = 6x^2 + 4x = 240$

$6x^2 + 4x - 240 = 0$

$3x^2 + 2x - 120 = 0$

$(3x+20)(x-6) = 0$

$x = \frac{-20}{3}$ | $x = 6$ ✓

7// (i) $N = N_0 e^{kt}$

- When $t = 0$, $N = 18$

$18 = N_0 e^{k(0)} \Rightarrow \underline{N_0 = 18}$

- November 1993, $t = 70$, $N = 5000$

$N = N_0 e^{kt}$

$5000 = 18 e^{k(70)}$

$e^{70k} = 277.78$

$\ln\left(e^{70k}\right) = \ln 277.78$

$70k = 5.63$

$k = 0.0803$

(ii) November 2023, $t = 100$

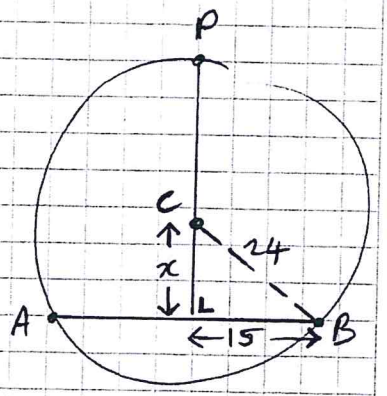
$N = N_0 e^{kt}$

$= 18 e^{0.0803(100)}$

$= 55,291.35$

$\simeq 55,292$

8//



$24^2 = x^2 + 15^2$

$576 = x^2 + 225$

$x^2 = 576 - 225$

$x^2 = 351$

$x = 3\sqrt{39}$

Height $= |PC| + |CD| + |DQ|$

$= 24 + 2(3\sqrt{39}) + 24$

$= 48 + 6\sqrt{39}$ cm