

5th Year Honours Maths - Problem Set 6

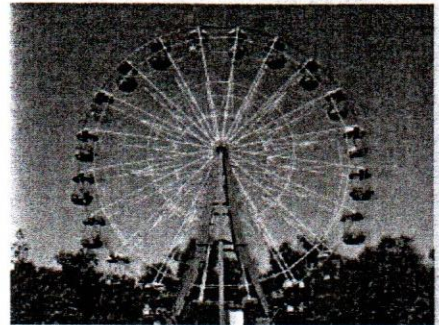
[Algebra, Trigonometry, Logs]

- Find the values of k for which the quadratic equation $3x^2 + 2x + k = 0$ has no real roots.
- If the value of a photocopier t years after purchase is given by $V(t) = 965 - 86t$
 - find $V(4)$ and state what $V(4)$ means
 - find t when $V(t) = 578$ and explain what this represents.
 - find the original purchase price of the photocopier.
- Sketch the curve $y = 3\sin 2x$ for $0 \leq x \leq 2\pi$. State the period and range of the graph.

- A Ferris wheel is turning at a steady rate. The height, h metres, of one of the cars above the ground at a time t seconds is given by the formula

$$h = 7 + 5\sin t^\circ.$$

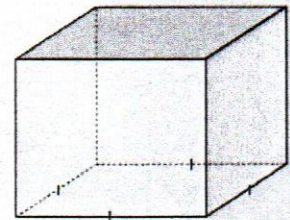
- Find the maximum and minimum heights of the car above the ground. [**Hint:** What are the max and min values of $\sin\theta$]
- Find **two** times during the first turn when the car is at a height of 10.8 metres above the ground.



- Solve the following equations:

(i) $\log_2(10x+7) - \log_2(x+1) = 1$ (ii) $\ln(4x+1) = 1.0986$ (iii) $3^x = 8$ (iv) $e^{2x+4} = 7.3890$

- A rectangular box has a square base and its height is 1cm longer than the length of one side of its base. If x cm is the length of one side of its base, show that its total surface area, A , is given by $A = 6x^2 + 4x$ cm².



If the total surface area is 240 cm², find the dimensions of the box.

- In November 1923, 18 koalas were introduced on Kangaroo Island. By November 1993, the number of koalas had increased to 5000. Assume that the number N of koalas is increasing exponentially and satisfies an equation of the form $N = N_0 e^{kt}$, where N_0 and k are constants and t is measured in years from November 1923.

- Find the values of N_0 and k .
- Predict the number of koalas that will be present on Kangaroo Island in November 2023.

- The shape below is used as a logo in an advertising campaign. It is made up from segments of two identical circles.

The points C and D are the centres of the circles and each circle has a radius of 24 centimetres. AB is a common chord of length 30 cm.

Calculate the height of the logo, represented by the line PQ .

