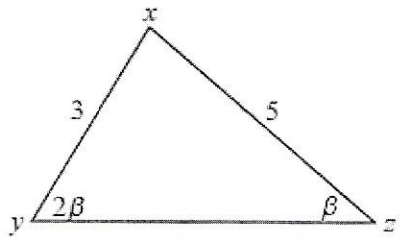
# 5th Year - Hons Maths. Problem Set 11

**1.** The cubic equation $x^3 + 4x^2 + x - 26 = 0$ has one integer root and 2 complex roots. Find all roots.

**2.** Show using the method of completing the square that $2x^2 + 5x - 7$ can be expressed as $\left(x + \frac{a}{b}\right)^2 - \frac{c}{d}$, where $a, b, c, d \in Z$.

**3.** The area of an equilateral triangle is $4\sqrt{3}$ cm$^2$. Find the length of a side of the triangle.

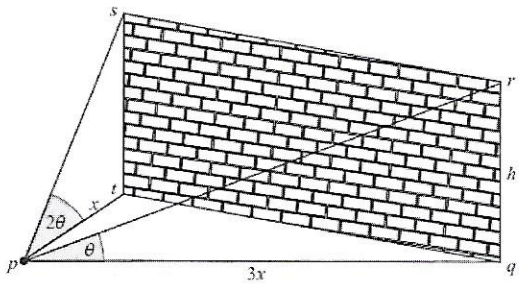
**4.** In the triangle $xyz$, $|\angle xyz| = 2\beta$ and $|\angle xzy| = \beta$, $|xy| = 3$ and $|xy| = 5$.




(i) Use this information to express $\sin 2\beta$ in the form $\frac{a}{b}\sin\beta$ where $a, b, c, d \in Z$

(ii) Hence express $\tan\beta$ in the form $\frac{\sqrt{c}}{d}$ where $c, d \in Z$.

**5.** The diagram shows a vertical rectangular wall *qrst* of height $h$ on level ground. $p$ is a point on the ground in front of the wall. The angle of elevation of $r$ from $p$ is $\theta$ and the angle of elevation of $s$ from $p$ is $2\theta$. Also it is given that $|pq| = 3|pt|$.



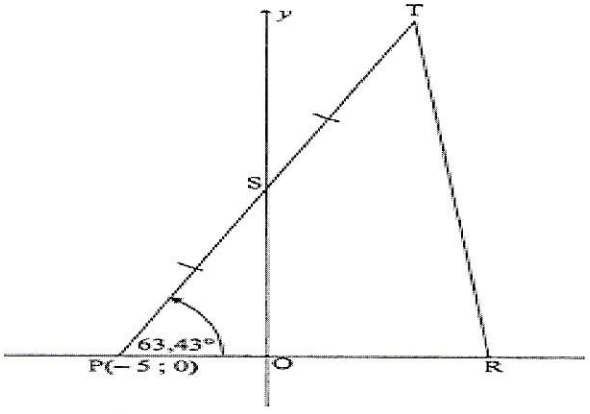

(i) Express $h$ in terms of $\tan\theta$ and $x$.
(ii) Express $h$ in terms of $\tan 2\theta$ and $x$.
(iii) Hence evaluate $\theta$.

**6.** The graph of $f(x) = ax^3 + bx^2 + cx + d$ crosses the $x-$axis at $x = 1$, $x = -2$ and $x = \frac{1}{2}$. It also crosses the y-axis at the point (0,6). Find the coefficients $a, b, c$ and $d$.

**7.** Simplify the following expressions giving your answer in the form $a + bi$ where $a, b \in R$.

(i) $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^4$ (ii) $\dfrac{\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}}{\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}}$

**8.** In the given diagram, the line PT makes an angle of 63.43° with the $x$-axis at the point P(-5,0). The line PT intersects the y-axis at S and $|PS| = |ST|$. The point R is on the $x$-axis such that $|PO| : |OR| = 2:3$. Find (i) the slope of PT to the nearest integer.




(ii) the equation of PT in the form $y = mx + c$

(iii) the distance PS in surd form.

(iv) the co-ordinates of T.

(v) the co-ordinates of R.

(vi) the area of the triangle PTR