Problem Set 11 - Solutions

1/ $f(x) = x^3 + 4x^2 + x - 26$

Trial + Error

$f(2) = (2)^3 + 4(2)^2 + 2 - 26$

$= 8 + 16 + 2 - 26$

$= 26 - 26$

$= 0$

$\Rightarrow x = 2$ is a root

$\Rightarrow x - 2$ is a factor

$$\begin{array}{r|l} & x^2 + 6x^2 + 13 \\ x - 2 & x^3 + 4x^2 + x - 26 \\ & \underline{x^3 - 2x^2} \\ & 6x^2 + x \\ & \underline{6x^2 - 12x} \\ & 13x - 26 \\ & \underline{13x - 26} \\ & 0 \quad 0 \end{array}$$

Now, Solve $x^2 + 6x + 13 = 0$

$a = 1,\ b = 6,\ c = 13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(13)}}{2}$$

$$x = \frac{-6 \pm \sqrt{-16}}{2} = \frac{-6 \pm 4i}{2}$$

$x = -3 + 2i,\ x = -3 - 2i$

Roots: $x = 2, -3 + 2i, -3 - 2i$

2/ $2x^2 + 5x - 7$

$= 2\left(x^2 + \frac{5}{2}x - \frac{7}{2}\right)$

$= 2\left(x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 - \frac{7}{2}\right)$

$= 2\left(\left(x + \frac{5}{2}\right)^2 - \frac{25}{16} - \frac{7}{2}\right)$

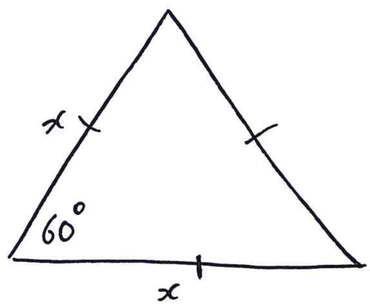
$= 2\left(\left(x + \frac{5}{2}\right)^2 - \frac{81}{16}\right)$

$= 2\left(x + \frac{5}{2}\right)^2 - \frac{81}{8}$

$k = 2,\ a = 5,\ b = 2$, etc

3/ Area $= 4\sqrt{3}$

$= \frac{1}{2} ab \sin C$



$\frac{1}{2}(x)(x) \sin 60° = 4\sqrt{3}$

$\frac{x^2}{2} \cdot \frac{\sqrt{3}}{2} = 4\sqrt{3}$

$x^2 = 16$

$x = 4$

4. Using Sine Rule

(i)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{3}{\sin \beta} = \frac{5}{\sin 2\beta}$$

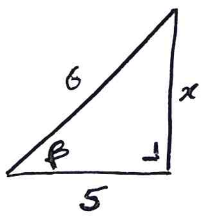
$$3 \sin 2\beta = 5 \sin \beta$$

$$\sin 2\beta = \frac{5}{3} \sin \beta$$

(ii) $\sin 2\beta = \frac{5}{3} \sin\beta$

$$2 \sin\beta \cos\beta = \frac{5}{3} \sin \beta$$

$$\cos \beta = \frac{5}{6}$$



$$x = \sqrt{11}$$

$$\tan \beta = \frac{\sqrt{11}}{5}$$

5. (i) $\tan\theta = \frac{h}{3x} \Rightarrow h = 3x \tan\theta$

(ii) $\tan 2\theta = \frac{h}{x} \Rightarrow h = x \tan 2\theta$

(iii) $h = 3x \tan\theta = x \tan 2\theta$

$$3 \tan\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$3\tan\theta(1-\tan^2\theta) = 2\tan\theta$$

$$3\tan\theta - 3\tan^3\theta = 2\tan\theta$$

$$3\tan^3\theta - \tan\theta = 0$$

$$\tan\theta(3\tan^2\theta - 1) = 0$$

$\tan\theta = 0$ | $3\tan^2\theta = 1$

θ not defined | $\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ$

6. $x = 1, -2$ and $\frac{1}{2}$ are roots

$\Rightarrow$ $x-1$, $x+2$, $2x-1$ are factors

$$f(x) = k\left[(x-1)(x+2)(2x-1)\right] = k\left[2x^3 + x^2 - 5x + 2\right]$$

$$(0,6) \Rightarrow f(0) = 6 \Rightarrow k\left[2(0)^3 + 0^2 - 5(0) + 2\right] = 6$$

$$\Rightarrow 2k = 6$$

$$k = 3$$

$$f(x) = 3\left[2x^3 + x^2 - 5x + 2\right]$$

$$f(x) = 6x^3 + 3x^2 - 15x + 6$$

7/ (i) $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)^4=\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\left(\cos\frac{8\pi}{3}+i\sin\frac{8\pi}{3}\right)$

$=\cos\frac{9\pi}{3}+i\sin\frac{9\pi}{3}=\cos 3\pi+i\sin 3\pi=-1+0i$

(ii) $\dfrac{\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}}{\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}}=\dfrac{\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}}{\cos\left(\frac{-\pi}{3}\right)+i\sin\left(\frac{-\pi}{3}\right)}$

$\left[\begin{array}{l}\cos(A)=\cos(-A)\\ -\sin(A)=\sin(-A)\end{array}\right]$

$=\cos\left(\frac{2\pi}{3}-\left(\frac{-\pi}{3}\right)\right)+i\sin\left(\frac{2\pi}{3}-\left(-\frac{\pi}{3}\right)\right)=\cos\pi+i\sin\pi=-1+0i$

8/ (i) slope $=\tan 63{\cdot}43^\circ=2$

(ii) $y-y_1=m(x-x_1)$

$(x_1,y_1)=(-5,0)$

$m=2$

$y-0=2(x+5)$

$y=2x+10$

(iii) $P(-5,0)$ $S(0,?)$

Co-ords of S

At y-axis, $x=0$

$y=2x+10$

$y=2(0)+10=10$

$S(0,10)$

$|PS|=\sqrt{(0+5)^2+(10-0)^2}$

$=\sqrt{125}$

(iv) Since $|PS|=|ST|$

$\Rightarrow$ S is the midpoint of [PT]

$\Rightarrow$ use central symmetry

$P(-5,0)\longrightarrow(0,10)\longrightarrow T(5,20)$

(v) $|PO|:|OR|=2:3$

$\dfrac{|PO|}{|OR|}=\dfrac{2}{3}\Leftrightarrow |OR|=\dfrac{3(5)}{2}=7{\cdot}5$

Co-ords of $R(7{\cdot}5,0)$

(vi) Area $=\frac{1}{2}$ base $\times\perp h$

$=\frac{1}{2}|PR|\times$ (y co-ord of T)

$=\frac{1}{2}(12{\cdot}5)(20)$

$=125$